Inefficiencies in Globalized Economies with Labor Market Frictions

Online Appendix

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A Complements on the decentralized economy

A.1 The workers' problem

The budget constraints are:

$$\begin{array}{rcll} (1+\tau_c)(C_{H,e}+(1+\tau_e)\phi C_{F,e}) &=& (1-\tau_w)wh+\pi+T & \text{if employed} \\ (1+\tau_c)(C_{H,u}+(1+\tau_e)\phi C_{F,u}) &=& (1-\tau_w)\widetilde{b}+\pi+T & \text{if unemployed} \end{array} \qquad \text{(A-1)}$$

A.1.1 Optimal choices within the aggregate consumption basket

The optimizing program of each agent employed and unemployed agent $z = e, u$, is the following:

$$\begin{array}{rl} \max\limits_{C_{H,z},C_{F,z}} & C_z = \dfrac{C_{H,x}^{\xi} C_{F,x}^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} \\ \text{s.t.} & PC_z = C_{H,z} + \phi(1+\tau_e)C_{F,z} \end{array}$$

The first-order conditions relative to the consumption of home and foreign goods lead to the following arbitrage condition, for $z = e, u$:

$$\frac{U'_{C_{F,z}}}{U'_{C_{H,z}}} = (1+\tau_e)\phi \;\Leftrightarrow\; \frac{1-\xi}{\xi}\frac{C_{H,z}}{C_{F,z}} = (1+\tau_e)\phi, \qquad \text{(A-2)}$$

Equivalently, we obtain, for $z = e, u$:

$$\begin{array}{rcl} C_{H,z} &=& \xi P C_z \\ (1+\tau_e)\phi C_{F,z} &=& (1-\xi)PC_z \\ P &=& [(1+\tau_e)\phi]^{1-\xi} \end{array} \qquad \text{(A-3)}$$

Defining $C_H = NC_{H,e} + (1-N)C_{H,u}$ and $C_F = NC_{F,e} + (1-N)C_{F,u}$ the total domestic consumption of Home and Foreign (imported) goods respectively, the optimal sharing rules between domestic and foreign consumption can be expressed at the aggregate level:

$$
\begin{aligned}
& C_H = \xi P C \\
& (1+\tau_e)\phi C_F = (1-\xi) P C \\
\Rightarrow \quad & C_H = (1+\tau_e)\phi C_F \frac{\xi}{1-\xi}
\end{aligned} \tag{A-4}
$$

A.1.2 Optimal choice of the aggregate consumption level

For an employee The program is:

$$
\begin{aligned}
& \max_{C_e} \ \mathcal{U}_e = C_e - \sigma_L \frac{h^{1+\eta}}{1+\eta} \\
\text{s.t.} \quad & (1+\tau_c) P C_e = (1-\tau_w) w h + \pi + T \qquad (\lambda_e)
\end{aligned}
$$

The first-order condition is:

$$
\lambda_e = \frac{1}{(1+\tau_c)P}
$$

For an unemployed worker The program is:

$$
\begin{aligned}
& \max_{C_u} \ \mathcal{U}_u = C_u \\
\text{s.t.} \quad & (1+\tau_c) P C_u = (1-\tau_w) \widetilde{b} + \pi + T \qquad (\lambda_u)
\end{aligned}
$$

The first-order condition is:

$$
\lambda_u = \frac{1}{(1+\tau_c)P}
$$

With a linear utility in consumption, we have the same λ whatever the employment status, such that there will be no difficulty when solving the match surplus in the context where we discard the large family assumption:

$$
\lambda_u = \lambda_e = \lambda = \frac{1}{(1+\tau_c)P} \tag{A-5}
$$

Obtaining the job surplus for a worker In term of utility, the job surplus of being employed is $\mathcal{S}_e$ defined as:

$$
\begin{aligned}
\mathcal{S}_e &= \mathcal{U}_e - \mathcal{U}_u \\
&= C_e - C_u - \sigma_L \frac{h^{1+\eta}}{1+\eta}
\end{aligned}
$$

From the agents' budget constraints (A-1), the gap between C_e and C_u is equal to:

$$C_e - C_u = \frac{1-\tau_w}{1+\tau_c}\left(\frac{wh-\widetilde{b}}{P}\right),$$

such that the expression of the job surplus becomes:

$$\mathcal{S}_e = \frac{1-\tau_w}{1+\tau_c}\left(\frac{wh-\widetilde{b}}{P}\right) - \sigma_L\frac{h^{1+\eta}}{1+\eta}$$

Turning to the monetary expression of the job surplus, defined as $\mathcal{V}_e = \frac{S_e}{\lambda}$ (with λ being independent of the employment status), we get, making use of Equation (A-5):

$$\mathcal{V}_e = (1-\tau_w)(wh-\widetilde{b}) - P(1+\tau_c)\sigma_L\frac{h^{1+\eta}}{1+\eta} \tag{A-6}$$

A.2 The firms' problem

Given the production function and the matching function (with the number of match equal to the employment level $M = N$ in this static setting) respectively given by:

$$Y = Ah^{\alpha}N \tag{A-7}$$

$$N = \chi V^{\psi}, \tag{A-8}$$

the profit expression is:

$$\pi = Ah^{\alpha}N - (1+\tau_f)whN + cN - \bar{\omega}V \tag{A-9}$$

Job posting condition: From Equation (A-9), we can derive the free-entry condition that summarizes the job-posting behavior of the firm:

$$\pi = 0 \quad \leftrightarrow \quad Ah^{\alpha}N - (1+\tau_f)whN + cN = \bar{\omega}V$$

that becomes:

$$\frac{\bar{\omega}}{\chi}V^{\psi-1} = Ah^{\alpha} + c - (1+\tau_f)wh \tag{A-10}$$

From this, we can show that the model features a share of wages in GDP less than unitary, starting from the free-entry condition (A-10). Multiplying the condition by $N = \chi V^{\psi}$ yields:

$$\bar{\omega}V = Ah^{\alpha}N + cN - (1+\tau_f)whN$$

Equivalently, it comes that:

$$wNh = \frac{ANh^{\alpha} - \overline{\omega}V + c}{1+\tau_f} \quad\Leftrightarrow\quad \frac{(1+\tau_f)wNh}{Y} = 1 - \frac{\overline{\omega}V - c}{Y}$$

This result demonstrates that the share of wages in GDP is smaller than 1 in the presence of non zero vacancy cost, even with a linear production function in N.

The job surplus for the firm. The surplus of a match for a firm, directly expressed in monetary terms, is given by

$$\mathcal{V}_f = \frac{\partial \pi}{\partial N}$$

with the expression of the profit given by Equation (A-9). We get:

$$\mathcal{V}_f = Ah^\alpha - (1+\tau_f)wh + c \tag{A-11}$$

A.3 Negotiating the match surplus

The hourly wage and the amount of hours worked per employee are the solutions of the bargaining problem:

$$\max_{w,h} \quad \Omega = \mathcal{V}_e^{1-\epsilon}\mathcal{V}_f^{\epsilon}$$

with $0 < \epsilon < 1$ the bargaining power of the firm in the negotiation. Making use of Equations (A-6) and (A-11), the problem rewrites:

$$\max_{w,h} \quad \Omega = \left[(1-\tau_w)(wh - \widetilde{b}) - P(1+\tau_c)\sigma_L \frac{h^{1+\eta}}{1+\eta}\right]^{1-\epsilon} \left[Ah^\alpha - (1+\tau_f)wh + c\right]^\epsilon$$

Obtaining the negotiated value for w: The first-order condition with respect to w gives:

$$\mathcal{V}_f = \frac{1+\tau_f}{1-\tau_w}\frac{\epsilon}{1-\epsilon}\mathcal{V}_e \tag{A-12}$$

Replacing $\mathcal{V}_e$ and $\mathcal{V}_f$ by the expressions from Equations (A-6) and (A-11), the bargained wage is given by:

$$wh = \frac{1-\epsilon}{1+\tau_f}\left(Ah^\alpha + c\right) + \frac{\epsilon}{1-\tau_w}\left((1-\tau_w)\widetilde{b} + P(1+\tau_c)\sigma_L\frac{h^{1+\eta}}{1+\eta}\right) \tag{A-13}$$

Obtaining the negotiated value for h: The first-order condition with respect to h gives:

$$(1-\epsilon)\left[(1-\tau_w)w - P(1+\tau_c)\sigma_L h^\eta\right] + \epsilon\frac{\mathcal{V}_e}{\mathcal{V}_f}\left[\alpha A h^{\alpha-1} - (1+\tau_f)w\right] = 0$$

Making use of Equation (A-12) to replace $\mathcal{V}_e$ and $\mathcal{V}_f$, and simplifying, we get that hours worked as set such as to equalize the marginal disutility of labor to the marginal productivity of labor, up to the tax wedge $TW \equiv \frac{(1+\tau_f)(1+\tau_c)}{1-\tau_w}$:

$$\sigma_L h^\eta P = \frac{1}{TW}\alpha A h^{\alpha-1} \tag{A-14}$$

A.4 Government and market equilibria

A.4.1 Government

In this static framework, the government budget constraint is necessarily balanced:

$$(1-\tau_w)\widetilde{b}(1-N) + cN = \tau_c\left[C_H + (1+\tau_e)\phi C_F\right] + \tau_e\phi C_F + (\tau_w+\tau_f)wNh + T, \tag{A-15}$$

where C_H and C_F represent the total domestic consumption of Home and Foreign (imported) goods respectively, and T denotes lump-sum taxes taken as exogenous. We assume that net unemployment benefits are proportional to the wage bill, that is, $\widetilde{b} = \rho_b wh$, with $0 < \rho_b < 1$; for analytical tractability reasons, we also assume a similar pattern for the employment subsidy ratio: $c = \rho_c(1+\tau_f)wh$, with $0 < \rho_c < 1$.

A.4.2 Market equilibria

Given the production function (A-7) and the foreign country's import demand function $Z^* = \phi^{\sigma^*}$, the home-goods equilibrium condition $Y = C_H + Z^* + \bar{\omega}V$ and the zero-trade balance equation $Z^* = \phi C_F$ can be rewritten as:

$$C_H = ANh^\alpha - \phi^{\sigma^*} - \overline{\omega}V, \tag{A-16}$$

$$C_F = \phi^{\sigma^*-1} \tag{A-17}$$

A.5 Solving the model

In this section, we detail the solving of the model in the decentralized case. The model's simplicity allows to solve it analytically. Precisely, it can be solved recursively until getting the equilibrium value of hours worked, from which we can deduce the equilibrium values for all macroeconomic variables.

Combining the optimal sharing rule (A-4) with the zero-trade balance equation (A-17), it comes:

$$\phi^{\sigma^*} = \frac{1-\xi}{\xi}\frac{C_H}{1+\tau_e}$$

$$\Leftrightarrow \qquad C_H = \frac{\xi}{1-\xi}(1+\tau_e)\phi^{\sigma^*}$$

Using this in the domestic good market equilibrium condition (A-16) (with $Y = ANh^\alpha$), we obtain:

$$Y - \overline{\omega}V = \frac{\xi}{1-\xi}(1+\tau_e)\phi^{\sigma^*} + \phi^{\sigma^*}$$

$$Y - \overline{\omega}V = \frac{\xi + (1-\xi)/(1+\tau_e)}{(1-\xi)/(1+\tau_e)}\phi^{\sigma^*}$$

$$\Rightarrow \qquad \phi = \left(\frac{(1-\xi)/(1+\tau_e)}{\xi + (1-\xi)/(1+\tau_e)}(Y - \overline{\omega}V)\right)^{\frac{1}{\sigma^*}} \tag{A-18}$$

with $1+t^e = \frac{1}{\xi+\frac{1-\xi}{1+\tau_e}}$, increasing with the tariff τ^e.

From this, we can also express the consumptions of the Home and Foreign goods in function of net output:

$$\begin{aligned} C_H &= \xi(1+t^e)(Y-\varpi V) && \text{(A-19)} \\ (1+\tau_e)\phi C_F &= (1-\xi)(1+t^e)(Y-\varpi V) && \text{(A-20)} \end{aligned}$$

Replacing ϕ by its above value, notice that we can express Home imports as a function of net output:

$$C_F = \left(\frac{(1-\xi)/(1+\tau_e)}{\xi+(1-\xi)/(1+\tau_e)}(Y-\varpi V)\right)^{\frac{\sigma^*-1}{\sigma^*}}$$

Free-entry condition Recall the job-posting free entry condition:

$$\frac{\bar{\omega}}{\chi}V^{1-\psi} - c = Ah^\alpha - (1+\tau_f)wh$$

Combining with the Nash solutions for w (A-13), it becomes:

$$\begin{aligned} \frac{\bar{\omega}}{\chi}V^{1-\psi} &= \epsilon\left[Ah^\alpha + c - b - TW\sigma_L\frac{h^{1+\eta}}{1+\eta}P\right] && \text{(A-21)} \\ \text{with} \quad & b \equiv (1+\tau_f)\tilde{b} \end{aligned}$$

Rewriting the condition on the hours worked (A-14) as follows:

$$\sigma_L\frac{h^{1+\eta}}{1+\eta}P = \frac{1}{TW}\frac{\alpha}{1+\eta}Ah^\alpha$$

the free-entry condition thus becomes:

$$\begin{aligned} &\frac{\varpi}{\chi}V^{1-\psi} = \epsilon\left[\frac{1+\eta-\alpha}{1+\eta}Ah^\alpha + c - b\right] \\ \Leftrightarrow \quad & V = (A\chi)^{\frac{1}{1-\psi}}h^{\frac{\alpha}{1-\psi}}\left[\frac{\epsilon}{\varpi}\left(\frac{1+\eta-\alpha}{1+\eta}+\frac{c-b}{Ah^\alpha}\right)\right]^{\frac{1}{1-\psi}} && \text{(A-22)} \end{aligned}$$

which we will make use of in stating the government's fiscal optimizing problem. Given that $N = \chi V^\psi$, one can rewrite the production function as:

$$\begin{aligned} Y &= (A\chi)h^\alpha V^\psi \\ &= (A\chi)^{\frac{1}{1-\psi}}h^{\frac{\alpha}{1-\psi}}\left[\frac{\epsilon}{\varpi}\left(\frac{1+\eta-\alpha}{1+\eta}+\frac{c-b}{Ah^\alpha}\right)\right]^{\frac{\psi}{1-\psi}} \end{aligned}$$

and the net output:

$$\begin{aligned} Y-\varpi V &= (A\chi)^{\frac{1}{1-\psi}}h^{\frac{\alpha}{1-\psi}}\left[\frac{\epsilon}{\varpi}\left(\frac{1+\eta-\alpha}{1+\eta}-\frac{b-c}{Ah^\alpha}\right)\right]^{\frac{\psi}{1-\psi}} - \varpi(A\chi)^{\frac{1}{1-\psi}}h^{\frac{\alpha}{1-\psi}}\left[\frac{\epsilon}{\varpi}\left(\frac{1+\eta-\alpha}{1+\eta}-\frac{b-c}{Ah^\alpha}\right)\right]^{\frac{1}{1-\psi}} \\ &= \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}h^{\frac{\alpha}{1-\psi}}\left[\frac{1}{\varpi}\left(\epsilon(1+\eta-\alpha)-\frac{\epsilon(1+\eta)(b-c)}{Ah^\alpha}\right)\right]^{\frac{\psi}{1-\psi}}\left[(1+\eta)-\epsilon(1+\eta-\alpha)+\frac{\epsilon(1+\eta)(b-c)}{Ah^\alpha}\right] \end{aligned} \quad \text{(A-23)}$$

Assuming that $c = \rho_c(1+\tau_f)wh$ at the equilibrium, and recalling that $b = (1+\tau_f)\widetilde{b} = (1+\tau_f)\rho_b wh$, this leads to:

$$Y - \overline{\omega}V = \Theta h^{\frac{\alpha}{1-\psi}}, \tag{A-24}$$

with Θ a function of deep parameters according to:

$$\Theta = \left(\frac{\chi A}{1+\eta}\right)^{\frac{1}{1-\psi}} \left(\frac{\epsilon}{\overline{\omega}} \frac{(1+\eta)(1-\rho_b) - \alpha(1-\rho_c)}{1-\epsilon\rho_b - \rho_c(1-\epsilon)}\right)^{\frac{\psi}{1-\psi}} [(1-\epsilon)(1+\eta)+\epsilon\alpha] \frac{1-\rho_c}{1-\epsilon\rho_b-\rho_c(1-\epsilon)} \tag{A-25}$$

Consider now the wage curve (A-13). Making use of $c = \rho_c(1+\tau_f)wh$ at the equilibrium, and $b = (1+\tau_f)\widetilde{b} = (1+\tau_f)\rho_b wh$, it can be rewritten as:

$$(1+\tau_f)wh = \frac{1}{1-\rho_b\epsilon-\rho_c(1-\epsilon)} Ah^{\alpha} \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1+\eta} \tag{A-26}$$

From this, we can deduce the following expression for gross unemployment benefits and the employment subsidy:

$$\begin{aligned}
b &= (1+\tau_f)\widetilde{b} = \rho_b(1+\tau_f)wh = \frac{\rho_b}{1-\rho_b\epsilon-\rho_c(1-\epsilon)} Ah^{\alpha}\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1+\eta} \\
c &= \rho_c(1+\tau_f)wh = \frac{\rho_c}{1-\rho_b\epsilon-\rho_c(1-\epsilon)} Ah^{\alpha}\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1+\eta} \\
\Rightarrow \frac{c-b}{Ah^{\alpha}} &= \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1+\eta}\left[\frac{\rho_c-\rho_b}{1-\rho_b\epsilon-\rho_c(1-\epsilon)}\right] \qquad \text{(A-27)}
\end{aligned}$$

Making use of Equation (A-27) in Equation (A-22), vacancies can be expressed as a function of hours worked according to:

$$V = \Theta^{\frac{1}{\psi}} \left[\frac{A\chi}{1+\eta}\right]^{-\frac{1}{\psi}} \left[(1-\rho_c)\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\rho_b\epsilon-\rho_c(1-\epsilon)}\right]^{-\frac{1}{\psi}} h^{\frac{\alpha}{1-\psi}}$$

Plugging Equation (A-18) in the CPI expression (A-3), this rewrites as:

$$P = (1+\tau_e)^{(1-\xi)} \left(\frac{(1-\xi)/(1+\tau_e)}{\xi + (1-\xi)/(1+\tau_e)}(Y-\overline{\omega}V)\right)^{\frac{1-\xi}{\sigma^*}} \tag{A-28}$$

Combining Equation (A-28) and net output given by Equation (A-24) in the negotiated value for hours worked (A-14), leads to the following solution for hours worked at the decentralized equilibrium:

$$h^{dec} = \left[\frac{A\alpha}{\sigma_L}\left[\frac{1}{1+\tau_e}\right]^{1-\xi} \frac{1}{TW}\left(\frac{1}{\frac{1-\xi}{1+\tau_e}(1+t^e)\Theta}\right)^{\frac{(1-\xi)}{\sigma^*}}\right]^{\nu} \tag{A-29}$$

$$\text{with} \qquad \begin{cases} 1+t^e \equiv \frac{1}{\xi+\frac{1-\xi}{1+\tau_e}} \\ \nu \equiv \frac{1-\psi}{(1+\eta-\alpha)(1-\psi)+\alpha\frac{1-\xi}{\sigma^*}} \end{cases}$$

and Θ defined in Equation (A-25).

From this, one can deduce the whole set of equilibrium values in the decentralized economy (suppressing the ^{dec} subscript for reading convenience). The equilibrium values for V, Y can thus be obtained recursively as follows:

$$\frac{\bar{\omega}}{\chi}V^{1-\psi} = \epsilon\left[\frac{1+\eta-\alpha}{1+\eta} + \frac{c-b}{Ah^{\alpha}}\right]Ah^{\alpha} \tag{A-30}$$

$$\text{with} \quad \frac{c-b}{Ah^{\alpha}} = \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1+\eta}\left(\frac{\rho_c-\rho_b}{1-\epsilon\rho_b-(1-\epsilon)\rho_c}\right) \tag{A-31}$$

(A-32)

implying that vacancies rewrite as a function of hours worked according to:

$$V = \Theta^{\frac{1}{\psi}}\left[\frac{A\chi}{1+\eta}\right]^{-\frac{1}{\psi}}\left[(1-\rho_c)\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\rho_b\epsilon-\rho_c(1-\epsilon)}\right]^{-\frac{1}{\psi}} h^{\frac{\alpha}{1-\psi}} \tag{A-33}$$

with Θ as defined in Equation (A-25). The equilibrium value for the negotiated wage can be derived from Equation (D-66).

We can then deduce the equilibrium values of the aggregate consumption of domestic and foreign goods as well as the relative price of imports:

$$C_H = \xi(1+t^e)(Y-\bar{\omega}V) \tag{A-34}$$

$$C_F = \left[\frac{1-\xi}{1+\tau_e}(1+t^e)(Y-\bar{\omega}V)\right]^{\frac{\sigma^\star-1}{\sigma^\star}} \tag{A-35}$$

$$\phi = \left[\frac{1-\xi}{1+\tau_e}(1+t^e)(Y-\bar{\omega}V)\right]^{\frac{1}{\sigma^\star}} \tag{A-36}$$

with net output a function of hours worked through Equation (A-36). We can also deduce the equilibrium values of consumption levels specific to the employment status from the optimal sharing rules (obtained in Section A.1):

$$\frac{1-\xi}{\xi}\frac{C_{H,e}}{C_{F,e}} = (1+\tau_e)\phi$$

$$\frac{1-\xi}{\xi}\frac{C_{H,u}}{C_{F,u}} = (1+\tau_e)\phi$$

Integrating this result in the budget constraint, we obtain, for the employee:

$$(1+\tau_c)\left(C_{H,e} + (1+\tau_e)\phi\frac{1-\xi}{\xi}\frac{C_{H,e}}{(1+\tau_e)\phi}\right) = (1-\tau_w)wh + \pi + T$$

$$(1+\tau_c)\left(C_{H,e} + \frac{1-\xi}{\xi}C_{H,e}\right) = (1-\tau_w)wh + \pi + T$$

$$\Rightarrow \quad \frac{1}{\xi}(1+\tau_c)C_{H,e} = (1-\tau_w)wh + \pi + T$$

Given that $\pi = 0$ in equilibrium, and given the exogenous values of T, τ_c and τ_w and the equilibrium values for h and w, this implicitly defines the equilibrium value for the consumption level of Home goods by the employed worker $C_{H,e}$:

$$C_{H,e} = \xi \frac{(1-\tau_w)wh + T}{1+\tau_c}$$

According to a similar reasoning, the consumption level for the Home good for the unemployed worker can be obtained through:

$$C_{H,u} = \xi \frac{(1-\tau_w)\widetilde{b} + T}{1+\tau_c}$$

The consumption of imported goods $C_{F,e}$, $C_{F,u}$ follow from the sharing rule (A-2). Notice that, as long as $\rho_b < 1 \leftrightarrow \widetilde{b} < wh$, consumption levels of the unemployed workers are lower than that of the employed.

Adopting an utilitarist view, aggregate welfare is defined as the weighted sum of utilities of each category of agents:

$$\begin{aligned} & \mathcal{U}^{dec} = N\mathcal{U}_e + (1-N)\mathcal{U}_u \\ \Leftrightarrow \quad & \mathcal{U}^{dec} = NC_e + (1-N)C_u - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \end{aligned}$$

With $PC_e = C_{H,e} + (1+\tau_e)\phi C_{F,e}$ and $PC_u = C_{H,u} + (1+\tau_e)\phi C_{F,u}$, aggregate welfare can be rewritten as:

$$\begin{aligned} \mathcal{U}^{dec} &= N\frac{C_{H,e} + (1+\tau_e)\phi C_{F,e}}{P} + (1-N)\frac{C_{H,u} + (1+\tau_e)\phi C_{F,u}}{P} - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \\ &= \frac{1}{P}\left[NC_{H,e} + (1-N)C_{H,u}\right] + \frac{\phi(1+\tau_e)}{P}\left[NC_{F,e} + (1-N)C_{F,u}\right] - N\sigma_L \frac{h^{1+\eta}}{1+\eta}, \end{aligned}$$

leading to:

$$\mathcal{U}^{dec} = \frac{1}{P}\left[C_H + (1+\tau_e)\phi C_F\right] - N\sigma_L \frac{h^{1+\eta}}{1+\eta}. \tag{A-37}$$

Positivity condition on hours worked For the decentralized equilibrium to exist, it is necessary to impose some extra condition to ensure a positive number of hours worked in equilibrium. From the above system this requires $\Theta > 0$. From Equation (A-25)), this is ensured under the twofold (sufficient) condition:

$$\begin{aligned} & 1 - \epsilon\rho_b - \rho_c(1-\epsilon) > 0 \\ \text{and} \quad & (1+\eta)(1-\rho_b) - \alpha(1-\rho_c) > 0 \end{aligned}$$

First consider the first condition $1 - \rho_b\epsilon - \rho_c(1-\epsilon) > 0$, which rewrites as:

$$\begin{aligned} & 1 - \rho_c(1-\epsilon) > \rho_b\epsilon \\ \Leftrightarrow \quad & \rho_b < \frac{1-\rho_c(1-\epsilon)}{\epsilon} \end{aligned}$$

Since $0 < \epsilon < 1$ and $0 \leq \rho_c < 1$, the term $\frac{1-\rho_c(1-\epsilon)}{\epsilon}$ lies within the range $[1; 1/\epsilon]$. Given the definition of $\rho_b < 1$, then the condition $\rho_b < \frac{1-\rho_c(1-\epsilon)}{\epsilon}$ is always fulfilled.

Consider now the second condition $(1+\eta)(1-\rho_b) - \alpha(1-\rho_c) > 0$, which rewrites as:

$$\rho_b < 1 - \frac{\alpha(1-\rho_c)}{1+\eta} < 1$$

Defining:

$$\overline{\rho} \equiv 1 - \frac{\alpha}{1+\eta}$$

the positivity condition rewrites as:

$$\rho_b < \overline{\rho} + \frac{\alpha\rho_c}{1+\eta} < 1$$

This condition ensures a positive number of hours worked in the decentralized equilibrium.

A.6 Summary

The model's main variables at the decentralized equilibrium can be summarized through the following system, that solved recursively (with ν, Θ, t_e combinations of the deep parameters as previously defined).

$$\begin{aligned}
h &= \left[\frac{A\alpha}{\sigma_L} \left[\frac{1}{1+\tau_e} \right]^{1-\xi} \frac{1}{TW} \left(\frac{1}{\frac{1-\xi}{1+\tau_e}(1+t^e)\Theta} \right)^{\frac{(1-\xi)}{\sigma^*}} \right]^{\nu} \\
V &= \Theta^{\frac{1}{\psi}} \left[\frac{A\chi}{1+\eta} \right]^{-\frac{1}{\psi}} \left[(1-\rho_c) \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\rho_b\epsilon-\rho_c(1-\epsilon)} \right]^{-\frac{1}{\psi}} h^{\frac{\alpha}{1-\psi}} \\
Y^{net} &\equiv Y - \bar{\omega}V = \Theta h^{\frac{\alpha}{1-\psi}} \\
\phi &= \left(\frac{1-\xi}{1+\tau_e}(1+t_e)\Theta \right)^{\frac{1}{\sigma^*}} h^{\frac{\alpha}{\sigma^*(1-\psi)}} \\
Y &= Y^{net} + \bar{\omega}V \\
C_H &= \xi(1+t_e)Y^{net} \\
C_F &= (1-\xi)\frac{1+t_e}{1+\tau_e} Y^{net} \\
P &= [(1+\tau_e)\phi]^{1-\xi} \\
\mathcal{U}^{dec} &= \frac{1}{P}\left[C_H + (1+\tau_e)\phi C_F \right] - N\sigma_L \frac{h^{1+\eta}}{1+\eta}
\end{aligned}$$

B Centralized economy

B.1 The planner's objective: Aggregation issue

Assuming an utilitarist view, the planner's objective is to maximize the following welfare function:

$$\begin{aligned}
& \mathcal{U}^{sp} && = N\mathcal{U}_e + (1-N)\mathcal{U}_u \\
\Leftrightarrow \quad & \mathcal{U}^{sp} && = NC_e + (1-N)C_u - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \\
& && = N\frac{C_{He}^{\xi}C_{Fe}^{-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} + (1-N)\frac{C_{Hu}^{\xi}C_{Fu}^{-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} - N\sigma_L\frac{h^{1+\eta}}{1+\eta} \qquad \text{(B-38)}
\end{aligned}$$

under the set of the technological constraint, the matching function, the market equilibria and the export demand function:

$$\begin{cases}
N = \chi V^{\psi} \\
Y = ANh^{\alpha} \\
Y = (NC_{He} + (1-N)C_{Hu}) + Z^* + \bar{\omega} \\
Z^* = \phi\,(NC_{Fe} + (1-N)C_{Fu}) \\
Z^* = \phi^{\sigma^*}
\end{cases} \qquad \text{(B-39)}$$

Consolidating the above system (B-39), the planners' program can be rewritten in a more compact way as:

$$\max_{\{C_{He},C_{Hu},C_{Fe},C_{Fu},h,V\}} \mathcal{U}^{sp} = N\frac{C_{He}^{\xi}C_{Fe}^{-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} + (1-N)\frac{C_{Hu}^{\xi}C_{Fu}^{-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} - N\sigma_l\frac{h^{1+\eta}}{1+\eta}$$

$s.t.$

$$N = \chi V^{\psi} \qquad \text{(B-40)}$$

$$A\chi V^{\psi}h^{\alpha} = NC_{He} + (1-N)C_{Hu} + \phi\sigma^* + \bar{\omega}V \qquad \text{(B-41)}$$

$$NC_{Fe} + (1-N)C_{Fu} = \phi^{\sigma^*-1} \qquad \text{(B-42)}$$

In a first step, we show that the planner's program yields to attribute the same amount of consumption of both Home and Foreign goods to each worker in the economy, whatever her employment status. Denoting λ_1, λ_2 and λ_3 the multipliers associated to each constraint (B-40), (B-41) and (B-42) respectively and forming the Lagrangian of the problem, one gets the following first-order conditions with respect to the consumption levels of the Home good, for the employed and unemployed worker respectively:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial C_{He}} = 0 \quad & \leftrightarrow \quad N\xi^{1-\xi}(1-\xi)^{-(1-\xi)}C_{He}^{\xi-1}C_{Fe}^{1-\xi} - N\lambda_1 = 0 \\
\frac{\partial\mathcal{L}}{\partial C_{Hu}} = 0 \quad & \leftrightarrow \quad (1-N)\xi^{1-\xi}(1-\xi)^{-(1-\xi)}C_{Hu}^{\xi-1}C_{Fu}^{1-\xi} - (1-N)\lambda_1 = 0
\end{aligned}$$

Both equations can be consolidated according to:

$$\lambda_1 = \left(\frac{\xi}{1-\xi}\right)^{1-\xi} C_{He}^{\xi-1} C_{Fe}^{1-\xi} = \left(\frac{\xi}{1-\xi}\right)^{1-\xi} C_{Hu}^{\xi-1} C_{Fu}^{1-\xi} \tag{B-43}$$

The FOCs with respect to the consumption levels of the Foreign goods are respectively:

$$\frac{\partial \mathcal{L}}{\partial C_{Fe}} = 0 \quad \leftrightarrow \quad N(1-\xi)^{\xi}\xi^{-\xi} C_{He}^{\xi} C_{Fe}^{-\xi} - N\lambda_2 = 0$$
$$\frac{\partial \mathcal{L}}{\partial C_{Fu}} = 0 \quad \leftrightarrow \quad (1-N)(1-\xi)^{\xi}\xi^{\xi} C_{Hu}^{\xi} C_{Fu}^{\xi} - (1-N)\lambda_2 = 0$$

Both equations can be consolidated according to:

$$\lambda_2 = \left(\frac{1-\xi}{\xi}\right)^{\xi} C_{He}^{\xi} C_{Fe}^{-\xi} = \left(\frac{1-\xi}{\xi}\right)^{\xi} C_{Hu}^{\xi} C_{Fu}^{-\xi} \tag{B-44}$$

Considering Equations (B-43) and (B-44 yields:

$$\frac{\lambda_1}{\lambda_2} = \frac{\xi}{1-\xi}\frac{C_{Fu}}{C_{Hu}} = \frac{\xi}{1-\xi}\frac{C_{Fe}}{C_{He}},$$

which implies:

$$\frac{C_{Fu}}{C_{Hu}} = \frac{C_{Fe}}{C_{He}} = \frac{C_F}{C_H}$$

In allocating consumptions across individuals, the Home planner does not make the distinction according to the employment status (playing so as a perfect insurance scheme), as $C_{Fe} = C_{Fu} = C_F$ and $C_{He} = C_{Hu} = C_H$. Further, the set of constraints (B-39) can also be rewritten through Equations (A-7) and (A-8) (technological constraints) and the market equilibrium conditions (A-16) and (A-17). Importantly, the Home planner takes into account the import demand function from the rest of the world related Home exports and the terms of trade as specified in Equation (A-17).

B.2 The planner's program

These results drive us to rewrite the welfare function of the planner (B-38) as:

$$\max_{C_H, C_F, h, N} \mathcal{U}^{sp} = \frac{C_H^{\xi} C_F^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \tag{B-45}$$

under the set of above constraints (B-39). Making use of this constraints set to replaceprivate consumptions C_H, C_F in the objective function (B-45), the problem is equivalent to choosing $\{\phi, h, V\}$ so as to maximize:

$$\max_{\phi, V, h} \mathcal{U}^{sp} = \max \left\{ \frac{(Y(h,V) - Z^*(\phi) - \varpi V)^{\xi} X^*(\phi)^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} - N(V)\sigma_L \frac{h^{1+\eta}}{1+\eta} \right\}$$

where Home import and exports volumes are respectively given by: $X^* = C_F = \phi^{\sigma^*-1}$ and $Z^* = \phi^{\sigma^*}$, output and employment by the technological constraints (A-7) and (A-8) as functions of hours worked and vacancies.

The first-order conditions with respect to ϕ, h and V are respectively:

$$\mathcal{U}_\phi^{sp\prime} = 0 \quad \leftrightarrow \quad \frac{U_{C_F}^{sp\prime}}{U_{C_H}} = \frac{\epsilon_{Z^*/\phi}}{\epsilon_{X^*/\phi}} \frac{Z^*}{X^*} \tag{B-46}$$

$$\mathcal{U}_h^{sp\prime} = 0 \quad \leftrightarrow \quad -\frac{U_{C_H}^{sp\prime}}{U_h} = Y_h' \tag{B-47}$$

$$\mathcal{U}_V^{sp\prime} = 0 \quad \leftrightarrow \quad U_{C_H}^{sp\prime}\left[Y_V' - \overline{\omega}\right] = N_V' \sigma_L \frac{h^{1+\eta}}{1+\eta} \tag{B-48}$$

with $\epsilon_{Z^*/\phi}$ the elasticity of foreign imports (i.e., home exports $X = Z^*$) and $\epsilon_{X^*/\phi}$ the elasticity of foreign exports (i.e., home imports) with respect to ϕ, and $\mu^* \equiv \frac{\sigma^*}{\sigma^*-1} > 1$.

Using the functional forms, we deduce that:

$$\begin{aligned}
\frac{1-\xi}{\xi}\frac{C_H}{C_F} &= \mu^*\phi \\
\alpha A h^\alpha &= \sigma_L h^{1+\eta} \frac{\xi^{\xi-1}(1-\xi)^{1-\xi}}{C_H^{\xi-1} C_F^{1-\xi}} \\
\frac{\overline{\omega}}{\chi} V^{1-\psi} &= \psi \left[A h^\alpha - \sigma_L \frac{h^{1+\eta}}{1+\eta} \frac{\xi^{\xi-1}(1-\xi)^{1-\xi}}{C_H^{\xi-1} C_F^{1-\xi}} \right],
\end{aligned}$$

leading to:

$$\begin{aligned}
\alpha A h^\alpha &= \sigma_L h^{1+\eta} (\mu^*\phi)^{1-\xi} \\
\frac{\overline{\omega}}{\chi} V^{1-\psi} &= \psi \frac{1+\eta-\alpha}{1+\eta} A h^\alpha
\end{aligned}$$

Remark that using $N = \chi V^\psi$ and the last equation, we obtain $Y - \overline{\omega} V = Y \frac{1+\eta-\psi(1+\eta-\alpha)}{1+\eta}$.

Replacing C_F through the zero-trade balance condition ($C_F = X^*(\phi) = \phi^{\sigma^*-1}$), we get C_H as a function of ϕ:

$$C_H = \frac{\xi}{1-\xi} \frac{\sigma^*}{\sigma^*-1} \phi^{\sigma^*} \tag{B-49}$$

The equilibrium on the home good market leads to $Y - \overline{\omega} V = C_H + Z^*(\phi) = C_H + \phi^{\sigma^*}$. Replacing C_H and C_F by their expressions in function of ϕ, we get:

$$\begin{aligned}
Y - \overline{\omega} V &= \frac{\xi}{1-\xi} \frac{\sigma^*}{\sigma^*-1} \phi^{\sigma^*} + \phi^{\sigma^*} \\
&= \frac{\xi + (1-\xi)\frac{\sigma^*-1}{\sigma^*}}{(1-\xi)\frac{\sigma^*-1}{\sigma^*}} \phi^{\sigma^*} \\
\Rightarrow \phi &= \left(\frac{(1-\xi)\frac{\sigma^*-1}{\sigma^*}}{\xi + (1-\xi)\frac{\sigma^*-1}{\sigma^*}} (Y - \overline{\omega} V) \right)^{\frac{1}{\sigma^*}}
\end{aligned}$$

Denoting

$$\mu^* = \frac{\sigma^*}{\sigma^* - 1}, \qquad \text{and} \qquad 1 + t^* = \frac{1}{\xi + \frac{1-\xi}{\mu^*}}$$

we can rewrite:

$$C_H = \xi(1+t^*)(Y - \varpi V) \tag{B-50}$$

$$C_F = \left[\frac{1-\xi}{\mu^*}(1+t^*)(Y-\varpi V)\right]^{\frac{\sigma^*-1}{\sigma^*}} \tag{B-51}$$

$$\phi = \left[\frac{1-\xi}{\mu^*}(1+t^*)(Y-\varpi V)\right]^{\frac{1}{\sigma^*}} \tag{B-52}$$

Combining Equations (B-47) and (B-48), we obtain the function that relates job vacancies to hours worked at the planner's solution:

$$V = (A\chi)^{\frac{1}{1-\psi}} h^{\frac{\alpha}{1-\psi}} \left(\frac{\psi}{\varpi}\frac{1+\eta-\alpha}{1+\eta}\right)^{\frac{1}{1-\psi}}$$

Combining this with the production function (A-7) and the matching function (A-8) leads to obtain output as a function of hours worked according to:

$$Y = (A\chi)^{\frac{1}{1-\psi}} \left(\frac{\psi}{\varpi}\frac{1+\eta-\alpha}{1+\eta}\right)^{\frac{\psi}{1-\psi}} h^{\frac{\alpha}{1-\psi}}$$

From which we can deduce net output as a function of hours worked:

$$Y - \varpi V = (A\chi)^{\frac{1}{1-\psi}} h^{\frac{\alpha}{1-\psi}} \left(\frac{\psi}{\varpi}\frac{1+\eta-\alpha}{1+\eta}\right)^{\frac{\psi}{1-\psi}} \left(\frac{1+\eta-\psi(1+\eta-\alpha)}{1+\eta}\right)$$

leading to:

$$Y - \varpi V = \Psi h^{\frac{\alpha}{1-\psi}} \tag{B-53}$$

$$\text{with} \qquad \Psi = \left(\frac{\chi A}{1+\eta}\right)^{\frac{1}{1-\psi}} \left(\frac{\psi}{\varpi}(1+\eta-\alpha)\right)^{\frac{\psi}{1-\psi}} [(1-\psi)(1+\eta)+\psi\alpha] \tag{B-54}$$

Combining the zero-trade balance condition $C_F = \phi^{\sigma^*-1}$ and the sharing rule (B-46), we can rewrite:

$$C_H^{\xi-1} C_F^{1-\xi} = \xi^{\xi-1}\left(\frac{\mu^*}{1-\xi}\right)^{\xi-1} \phi^{\xi-1}$$

Making use of this, Equation (B-47) simplifies to get:

$$\sigma_L h^{1+\eta-\alpha} = A\alpha(\mu^*\phi)^{\xi-1}$$

Using the solution for ϕ given by Equation (B-52), this rewrites as:

$$h = \left[\frac{\alpha A}{\sigma_L} (\mu^*)^{\xi-1} \left(\frac{\frac{1-\xi}{\mu^*}}{\xi + \frac{1-\xi}{\mu^*}} \right)^{\frac{\xi-1}{\sigma^*}} \right]^{\frac{1}{1+\eta-\alpha}} (Y - \bar{\omega} V)^{\frac{\xi-1}{\sigma^*(1+\eta-\alpha)}}$$

Replacing $Y - \bar{\omega} V$ through Equation (B-53) leads to:

$$h = \left[\frac{\alpha A}{\sigma_L} (\mu^*)^{\xi-1} \left(\frac{\frac{1-\xi}{\mu^*}}{\xi + \frac{1-\xi}{\mu^*}} \right)^{\frac{\xi-1}{\sigma^*}} \Psi^{\frac{\xi-1}{\sigma^*}} \right]^{\frac{1-\psi}{(1-\psi)(1+\eta-\alpha)+\alpha\frac{1-\xi}{\sigma^*}}}$$

Denoting $1 + t^* = \frac{1}{\xi + \frac{1-\xi}{\mu^*}}$, the equilibrium value of hours worked at the planner's solution can be expressed as:

$$h^{sp} = \left[\frac{\alpha A}{\sigma_L} \left(\frac{1}{\mu^*} \right)^{1-\xi} \left(\frac{1}{\frac{1-\xi}{\mu^*}(1+t^*)\Psi} \right)^{\frac{1-\xi}{\sigma^*}} \right]^{\nu}$$

with ν similarly defined as in the decentralized case and Ψ as defined above.

B.3 Summary

The model's main variables at the planner's solution can be summarized through the following system, that solved recursively (with ν, Ψ, μ^*, t^* combinations of the deep parameters as previously defined).

$$\begin{aligned}
h &= \left[\frac{A\alpha}{\sigma_L} \left[\frac{1}{\mu^*} \right]^{1-\xi} \left(\frac{1}{\frac{1-\xi}{\mu^*}(1+t^*)\Psi} \right)^{\frac{(1-\xi)}{\sigma^*}} \right]^{\nu} \\
V &= \Psi^{\frac{1}{\psi}} \left[\frac{A\chi}{1+\eta} \right]^{-\frac{1}{\psi}} [(1+\eta)(1-\psi) + \psi\alpha]^{-\frac{1}{\psi}} h^{\frac{\alpha}{1-\psi}} \\
Y^{net} &\equiv Y - \bar{\omega} V = \Psi h^{\frac{\alpha}{1-\psi}} \\
\phi &= \left(\frac{1-\xi}{\mu^*}(1+t^*)\Psi \right)^{\frac{1}{\sigma^*}} h^{\frac{\alpha}{\sigma^*(1-\psi)}} \\
Y &= Y^{net} + \bar{\omega} V \\
C_H &= \xi(1+t^*) Y^{net} \\
C_F &= (1-\xi) \frac{1+t^*}{\mu^*} Y^{net} \\
\mathcal{U}^{sp} &= \frac{C_H^{\xi} C_F^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}} - N\sigma_L \frac{h^{1+\eta}}{1+\eta}
\end{aligned}$$

C Identifying the inefficiency gaps

C.1 Inefficiency on hours worked

In this section, we identify the conditions under which the amount of hours worked at the decentralized equilibrium is inefficiently high, that is, higher than at the central planner's equilibrium. Throughout this section, we discard the presence of employment subsidy and trade taxes, i.e. imposing $\rho_c = \tau_e = 0$.

From Equation (A-29), it is straightforward that hours worked are a decreasing function of Θ. From Equations (A-25) and (B-54), the ratio $\frac{\Theta}{\Psi}$ is equal to (under $\rho_c = 0$):

$$\frac{\Theta}{\Psi} = \left[\frac{\epsilon}{\psi}\frac{(1+\eta)(1-\rho_b)-\alpha}{(1-\epsilon\rho_b)(1+\eta-\alpha)}\right]^{\frac{\psi}{1-\psi}}\left[\frac{1}{1-\epsilon\rho_b}\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{(1-\psi)(1+\eta)+\psi\alpha}\right]$$

The objective is to determine the conditions under which $\frac{\Theta}{\Psi} < 1$, implying $\frac{h^{dec}}{h^{sp}} > 1$.

Case 1: Where $\epsilon = \psi$. In this case, the ratio $\frac{\Theta}{\Psi}$ rewrites as:

$$\frac{\Theta}{\Psi} = \left[\frac{(1+\eta)(1-\rho_b)-\alpha}{(1-\psi\rho_b)(1+\eta-\alpha)}\right]^{\frac{\psi}{1-\psi}}\left[\frac{1}{1-\psi\rho_b}\right]$$

Considering the first term into bracket $\frac{(1+\eta)(1-\rho_b)-\alpha}{(1-\psi\rho_b)(1+\eta-\alpha)}$, it is equal to 1 as long as $\rho_b = 0$, in which case $\Theta = \Psi$. By contrast, under $\rho_b > 0$, given $\psi < 1$, we have: $\psi\rho_b < \rho_b$. Further, given $0 < \alpha < 1$, we have $1+\eta-\alpha < 1+\eta$. Accordingly,

$$\begin{aligned}
& \psi\rho_b(1+\eta-\alpha) < \rho_b(1+\eta) \\
\Leftrightarrow \quad & 1+\eta-\alpha 1-\psi\rho_b(1+\eta-\alpha) > 1+\eta-\alpha-\rho_b(1+\eta) \\
\Leftrightarrow \quad & \frac{1}{(1+\eta-\alpha)(1-\psi\rho_b)} < \frac{1}{1+\eta-\alpha-\rho_b(1+\eta)}
\end{aligned}$$

such that, under $\rho_b < \overline{\rho}$:

$$\frac{1+\eta-\alpha-\rho_b(1+\eta)}{(1+\eta-\alpha)(1-\psi\rho_b)} < 1 \quad \leftrightarrow \frac{\Theta}{\Psi} < 1$$

This establishes that, under $\epsilon = \psi$, having $\rho_b > 0$ is a sufficient condition to ensure $\frac{\Theta}{\Psi} < 1$.

Case 2: Where $\rho_b = 0$. In this case, the ratio $\frac{\Theta}{\Psi}$ simplifies into:

$$\frac{\Theta}{\Psi} = \left[\frac{\epsilon}{\psi}\right]^{\frac{\psi}{1-\psi}}\left[\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{(1-\psi)(1+\eta)+\psi\alpha}\right]$$

It is straightforward that having $\epsilon < \psi$ is sufficient to ensure $\frac{\Theta}{\Psi} < 1$.

General case We now prove the conditions under which $\frac{\partial\Theta}{\partial\rho_b} < 0$ and under which $\frac{\partial\Theta}{\partial\epsilon} > 0$. We obtain that Θ is monotonically decreasing with ρ_b and increasing with ϵ for $0 < \rho_b < \overline{\rho}$ provided $\epsilon < \underline{\epsilon}$, with $\underline{\epsilon} \equiv \frac{\psi}{\overline{\rho}-\rho_b(1-\psi)}$. In particular, $\frac{\partial\Theta}{\partial\rho_b} < 0$ and $\frac{\partial\Theta}{\partial\epsilon} > 0$ under the sufficient condition $\epsilon \leq \psi$ (and provided $\rho_b < \overline{\rho}$).

C.1.1 Establishing the derivative with respect to ρ_b

Deriving the above expression (A-25) with respect to ρ_b leads to:

$$\begin{aligned}\frac{\partial\Theta}{\partial\rho_b} &= \Upsilon\left(\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\epsilon\rho_b}\right)\left[\frac{\psi}{1-\psi}\frac{1-\epsilon\rho_b}{(1+\eta)(1-\rho_b)-\alpha}\right]\left(\frac{-(1+\eta)(1-\epsilon\rho_b)+\epsilon((1+\eta)(1-\rho_b)-\alpha)}{(1-\epsilon\rho_b)^2}\right)\\ &\quad+\Upsilon\left((1-\epsilon)(1+\eta)+\epsilon\alpha\right)\left(\frac{\epsilon}{(1-\epsilon\rho_b)^2}\right)\end{aligned}$$

with

$$\Upsilon \equiv \left[\frac{A\chi}{1+\eta}\right]^{\frac{1}{1-\psi}}\left[\frac{\epsilon}{\bar{\omega}}\frac{(1+\eta)(1-\rho_b)-\alpha}{1-\epsilon\rho_b}\right]^{\frac{\psi}{1-\psi}} \lesseqgtr 0$$

.

Simplifying:

$$\frac{\partial\Theta}{\partial\rho_b} = \Upsilon\frac{1}{(1-\epsilon\rho_b)^2}\left[\frac{\psi}{1-\psi}\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{(1+\eta)(1-\rho_b)-\alpha}\Upsilon_1+\epsilon\left((1+\eta)(1-\epsilon)+\epsilon\alpha\right)\right]$$

with $\Upsilon_1 = -(1+\eta)(1-\epsilon\rho_b)+\epsilon(1+\eta)(1-\rho_b)-\epsilon\alpha)$. One can rewrite Υ_1 as:

$$\Upsilon_1 = -\left[(1+\eta)(1-\epsilon)+\epsilon\alpha\right]$$

such that $\frac{\partial\Theta}{\partial\rho_b}$ becomes equal to:

$$\frac{\partial\Theta}{\partial\rho_b} = -\Upsilon\frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1-\epsilon\rho_b)^2}\left[\underbrace{\frac{\psi}{1-\psi}\left(\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{(1+\eta)(1-\rho_b)-\alpha}\right)-\epsilon}_{\Upsilon_2}\right]$$

Rewriting Υ_2, we get:

$$\Upsilon_2 = \frac{(\psi-\epsilon)(1+\eta)+\epsilon(1-\psi)\rho_b(1+\eta)+\epsilon\alpha}{(1-\psi)\left\{(1+\eta)(1-\rho_b)-\alpha\right\}}$$

Coming back to $\frac{\partial\Theta}{\partial\rho_b}$:

$$\frac{\partial\Theta}{\partial\rho_b} = -\Upsilon\frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1-\epsilon\rho_b)^2}\Upsilon_2$$

with:

$$\Upsilon = \left[\frac{A\chi}{1+\eta}\right]^{\frac{1}{1-\psi}}\left[\frac{\epsilon}{\bar{\omega}}\frac{(1+\eta)(1-\rho_b)-\alpha}{1-\epsilon\rho_b}\right]^{\frac{\psi}{1-\psi}}$$

$$\Upsilon_2 = \frac{(\psi-\epsilon)(1+\eta)+\epsilon(1-\psi)\rho_b(1+\eta)+\epsilon\alpha}{(1-\psi)\left\{(1+\eta)(1-\rho_b)-\alpha\right\}}$$

Reorganizing terms, it can be rewritten as:

$$\frac{\partial\Theta}{\partial\rho_b} = -\underline{\Theta}\left[\underbrace{(1+\eta)(1-\rho_b)-\alpha}_{\gamma_1}\right]^{-\frac{1-2\psi}{1-\psi}}\underbrace{\left[(\psi-\epsilon)(1+\eta)+\epsilon(1-\psi)\rho_b(1+\eta)+\epsilon\alpha\right]}_{\gamma_2}$$

with

$$\underline{\Theta} = \frac{(1+\eta)(1-\epsilon)+\epsilon\alpha}{(1-\psi)(1-\epsilon\rho_b)^2}\left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\left[\frac{\epsilon}{\bar{\omega}(1(\epsilon\rho_b)}\right]^{\frac{\psi}{1+\psi}} > 0$$

From this, the sign $\frac{\partial\Theta}{\partial\rho_b} < 0$ is ensured under two cases: ($\gamma_1 > 0$ and $\gamma_2 > 0$), or ($\gamma_1 < 0$ and $\gamma_2 < 0$).

Case 1: $\gamma_1 > 0$ and $\gamma_2 > 0$. From the expression of γ_1, we can state that $\gamma_1 > 0$ iif:

$$\begin{aligned} & (1+\eta)(1-\rho_b)-\alpha > 0 \\ \leftrightarrow \quad & \rho_b < \overline{\rho} \equiv 1 - \frac{1-\alpha}{1+\eta} \end{aligned}$$

with $\overline{\rho} < 1$. Notice this matches the positivity condition to ensure a positive number of hours worked at the decentralized equilibrium. Put it differently, it means that, provided a positive number of hours worked at the decentralized equilibrium, γ_1 is always positive. As a result, this rules out Case 2 ($\gamma_1 < 0$ and $\gamma_2 < 0$) as we briefly show below.

The positivity condition on γ_2 is ensured iif:

$$(\psi-\epsilon)(1+\eta)+\epsilon(1-\psi)\rho_b(1+\eta)+\epsilon\alpha > 0$$

that is:

$$\begin{aligned} & (\psi-\epsilon)(1+\eta)+\epsilon(1-\psi)\rho_b(1+\eta)+\epsilon\alpha > 0 \\ \Leftrightarrow \quad & \psi(1+\eta)-\epsilon(1+\eta)+\epsilon\left[(1-\psi)\rho_b(1+\eta)+\alpha\right] > 0 \\ \Leftrightarrow \quad & \epsilon\left[1-\rho_b(1-\psi)-\frac{\alpha}{1+\eta}\right] < \psi \\ \Leftrightarrow \quad & \epsilon\left[1-\rho_b-\frac{\alpha}{1+\eta}+\rho_b\psi\right] < \psi \\ \Leftrightarrow \quad & \epsilon\left[\overline{\rho}-\rho_b+\rho_b\psi\right] < \psi \end{aligned}$$

Provided $\rho_b < \overline{\rho}$, such that $\overline{\rho} - \rho_b + \rho_b\psi > 0$, then the above condition can be stated as putting an upward threshold value on ϵ:

$$\epsilon < \bar{\epsilon} \equiv \frac{\psi}{\overline{\rho} - \rho_b(1-\psi)}$$

Importantly, we can establish that $\bar{\epsilon} > \psi$. Indeed, $\bar{\epsilon} > \psi$ holds provided the denominator is superior to 1, i.e. $\overline{\rho} - \rho_b + \psi\rho_b > 1$. Given the definition of $\overline{\rho}$, this amounts having:

$$\begin{aligned} & 1 - \frac{\alpha}{1+\eta} - \rho_b(1-\psi) < 1 \\ \leftrightarrow \quad & \underbrace{\frac{1-\alpha+\eta}{1+\eta}}_{<1} - \underbrace{\rho_b(1-\psi)}_{<1} < 1 \end{aligned}$$

which is always the case. Alternatively, starting from the inequality condition $1 > \overline{\rho} > \rho_b$:

$$\begin{aligned} & 1 > \overline{\rho} > \rho_b \\ \leftrightarrow \quad & 1 - \rho_b > \overline{\rho} - \rho_b > 0 \\ \leftrightarrow \quad & 1 - \rho_b + \psi\rho_b > \overline{\rho} - \rho_b + \psi\rho_b \end{aligned}$$

neglecting the second part of the inequality as irrelevant here. The above inequality condition then rewrites as:

$$1 - \rho_b(1-\psi) > \overline{\rho} - \rho_b(1-\psi)$$

which necessarily implies that:

$$\overline{\rho} - \rho_b(1-\psi) < 1$$

As a result, it is always the case that $\bar{\epsilon} > \psi$.

Case 2: $\gamma_1 < 0$ and $\gamma_2 < 0$. From the expression of γ_1, we have $\gamma_1 < 0$ iif:

$$\begin{aligned} & (1+\eta)(1-\rho_b) - \alpha < 0 \\ \leftrightarrow \quad & \rho_b > \overline{\rho} \equiv 1 - \frac{\alpha}{1+\eta} \end{aligned}$$

Given that $\rho < 1$, this amounts having $\overline{\rho} < \rho_b < 1$, a configuration we have excluded to ensure a positive number of hours worked at the decentralized equilibrium. As a consequence, this eliminates Case 2 ($\gamma_1 < 0$ and $\gamma_2 < 0$) from the analysis.

Summary The only admissible case for $\frac{\partial\Theta}{\partial\rho_b} < 0$ is $\gamma_1 > 0$ and $\gamma_2 > 0$. From our above calculus, the necessary and sufficient condition for having $\gamma_1 > 0$ is $0 \leq \rho_b < \overline{\rho}$ with $\overline{\rho} \equiv 1 - \frac{\alpha}{1+\eta} < 1$; the necessary and sufficient condition for having $\gamma_2 > 0$ is twofold: $\rho_b < \overline{\rho}$ and $\epsilon < \bar{\epsilon}$, with $\bar{\epsilon} = \frac{\psi}{[\overline{\rho}-\rho_b(1-\psi)]} > \psi$. Accordingly, provided $0 \leq \rho_b < \overline{\rho}$ and $0 < \epsilon < \bar{\epsilon}$, then $\frac{\partial\Theta}{\partial\rho_b} < 0$ in which case $\frac{\partial h^{dec}}{\partial\rho_b} > 0$.

From this, we can establish the sufficient condition for $\frac{\partial\Theta}{\partial\rho_b} < 0$ as follows:

$$0 \leq \rho_b < \overline{\rho} \;\text{ and }\; \epsilon \leq \psi$$

In this case, the first part of the condition ensures $\gamma_1 > 0$; conditional on this, the second part of the condition $\epsilon \leq \psi$ is sufficient to ensure $\gamma_2 > 0$, since $\psi < \bar{\epsilon}$. Also notice that the condition $0 < \rho_b < \overline{\rho}$ or the condition $\epsilon < \psi$ are sufficient conditions to ensure $h^{dec} > h^{sp}$.

C.1.2 Establishing the derivative with respect to ϵ

Deriving the expression for Θ given by Equation (A-25) with respect to ϵ leads to:

$$\begin{aligned}
\frac{\partial\Theta}{\partial\epsilon} &= \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\left[\frac{\psi}{1-\psi}\widetilde{\Upsilon}^{\frac{\psi}{1-\psi}}\frac{\gamma_3}{\widetilde{\Upsilon}}\frac{\partial\widetilde{\Upsilon}}{\partial\epsilon} + \widetilde{\Upsilon}^{\frac{\psi}{1-\psi}}\frac{\partial\gamma_3}{\partial\epsilon}\right] \\
\text{with:} & \\
\widetilde{\Upsilon} &= \frac{\epsilon}{\bar{\omega}}\left(\frac{(1+\eta)(1-\rho_b)-\alpha}{1-\epsilon\rho_b}\right) \lesseqgtr 0 \\
\gamma_3 &= \frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{1-\epsilon\rho_b} > 0
\end{aligned}$$

This leads to:

$$\begin{aligned}
\frac{\partial\Theta}{\partial\epsilon} = & \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\widetilde{\Upsilon}^{\frac{\psi}{1-\psi}}\left\{\frac{\psi}{1-\psi}\frac{(1-\epsilon)(1+\eta)+\epsilon\alpha}{\epsilon}\left[\frac{1}{1-\epsilon\rho_b} + \epsilon\rho_b\frac{1}{(1-\epsilon\rho_b)^2}\right]\right\} \\
& + \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\widetilde{\Upsilon}^{\frac{\psi}{1-\psi}}\left\{\frac{1}{(1-\epsilon\rho_b)^2}\left\{-(1-\epsilon\rho_b)(1+\eta-\alpha)+\rho_b\left[(1-\epsilon)(1+\eta)+\epsilon\alpha\right]\right\}\right\}
\end{aligned}$$

Simplifying, we get:

$$\frac{\partial\Theta}{\partial\epsilon} = \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}}\widetilde{\Upsilon}^{\frac{\psi}{1-\psi}}\frac{1}{(1-\epsilon\rho_b)^2}\left\{\begin{array}{l} \frac{\psi}{\epsilon}\frac{1}{1-\psi}\left((1-\epsilon)(1+\eta)+\epsilon\alpha\right) \\ +\underbrace{\rho_b\left((1-\epsilon)(1+\eta)+\epsilon\alpha\right)-(1-\epsilon\rho_b)(1+\eta-\alpha)}_{\gamma_4} \end{array}\right\} \tag{C-55}$$

Let us consider the last term of the bracket:

$$\begin{aligned}
\gamma_4 &= \rho_b\left((1-\epsilon)(1+\eta)+\epsilon\alpha\right)-(1-\epsilon\rho_b)(1+\eta-\alpha) \\
&= \rho_b(1+\eta)-\epsilon\rho_b(1+\eta)-(1+\eta-\alpha)+\rho_b(1+\eta) \\
&= -(1+\eta)(1-\rho_b)+\alpha
\end{aligned}$$

Replacing γ_3 with the above expression in Equation (C-55):

$$\frac{\partial \Theta}{\partial \epsilon} = \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}} \widetilde{\Upsilon}^{\frac{\psi}{1-\psi}} \frac{1}{(1-\epsilon\rho_b)^2} \left\{ \begin{array}{l} \frac{\psi(1-\epsilon)(1+\eta)+\psi\epsilon\alpha}{\epsilon(1-\psi)} \\ + \frac{\epsilon(1-\psi)\alpha-\epsilon(1-\psi)(1+\eta)(1-\rho_b)}{\epsilon(1-\psi)} \end{array} \right\} \quad \text{(C-56)}$$

$$= \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}} \widetilde{\Upsilon}^{\frac{\psi}{1-\psi}} \frac{1}{(1-\epsilon\rho_b)^2} \frac{1}{\epsilon(1-\psi)} \underbrace{\left\{ \begin{array}{l} \psi(1-\epsilon)(1+\eta)+\psi\epsilon\alpha \\ +\epsilon(1-\psi)\alpha-\epsilon(1-\psi)(1+\eta)(1-\rho_b) \end{array} \right\}}_{\gamma_5} \quad \text{(C-57)}$$

Simplifying the term γ_5:

$$\begin{aligned} \gamma_5 &= \psi(1-\epsilon)(1+\eta)+\psi\epsilon\alpha+\epsilon(1-\psi)\alpha-\epsilon(1-\psi)(1+\eta)(1-\rho_b) \\ &= (1+\eta)\left[\psi(1-\epsilon)-\epsilon(1-\psi)\right]+\epsilon\left[\rho_b(1-\psi)(1+\eta)+\alpha\right] \\ &= (1+\eta)(\psi-\epsilon)+\epsilon\left[\rho_b(1-\psi)(1+\eta)+\alpha\right] \end{aligned}$$

Making use of this in Equation (C-57) and rearranging terms, we finally obtain:

$$\frac{\partial \Theta}{\partial \epsilon} = \bar{\Theta}\gamma_1^{\frac{\psi}{1-\psi}}\gamma_2$$

$$\text{with} \quad \left\{ \begin{array}{lcl} \bar{\Theta} &=& \left(\frac{A\chi}{1+\eta}\right)^{\frac{1}{1-\psi}} \frac{1}{(1-\epsilon\rho_b)^2} \frac{1}{\epsilon(1-\psi)} \left(\frac{\epsilon}{\bar{\omega}}\right)^{\frac{\psi}{1-\psi}} > 0 \\ \gamma_1 &=& (1+\eta)(1-\rho_b)-\alpha \\ \gamma_2 &=& (\psi-\epsilon)(1+\eta)+\epsilon\left[\rho_b(1-\psi)(1+\eta)+\alpha\right] \end{array} \right.$$

where we recognize in γ_1 and γ_2 the combinations of parameters already defined above. Consequently, the condition for $\frac{\partial \Theta}{\partial \epsilon} > 0$ is the same as for $\frac{\partial \Theta}{\partial \rho_b} < 0$: either ($\gamma_1 > 0$ and $\gamma_2 > 0$) or ($\gamma_1 < 0$ and $\gamma_2 < 0$). The same results apply accordingly.

D Optimal policy in a second-best world

In this section, we study the Ramsey policy in the decentralized setting where neither trade taxes nor the employment subsidy are available options to the government, whose sole instrument is the tax wedge (with $\tau_e = \rho_c = 0$). We also assume the positivity condition on hours worked holds (with $\rho_b < \bar{\rho}$).

D.1 Obtaining the objective of the government

The objective of the utilitarian government is to maximize the aggregate welfare of the decentralized economy :

$$\max_{TW} \mathcal{U}^g = N\mathcal{U}_e + (1-N)\mathcal{U}_u \equiv \mathcal{U}^{dec},$$

with $\mathcal{U}^{dec}$ as defined by Equation (A-37). From Equations (A-19) and (A-20) under $\tau_e = 0$, such that $\phi C_F = (1-\xi)(Y - \bar{\omega}V)$, the government's objective becomes:

$$\mathcal{U}^g = \frac{Y - \bar{\omega}V}{P} - N\sigma_L \frac{h^{1+\eta}}{1+\eta} \tag{D-58}$$

subject to the job-posting condition and the bargained solution for hours worked:

$$\begin{cases} \frac{\bar{\omega}V^{1-\psi}}{\chi} = \epsilon \left[\frac{1+\eta-\alpha}{1+\eta} Ah^\alpha - b\right] \\ \sigma_L h^{1+\eta} P = \frac{1}{TW} A\alpha h^\alpha \end{cases}$$

Integrating the technological constraints (A-8) and (A-7) and recalling that, from the decentralized equilibrium, we have:

$$\begin{aligned} P &= [(1-\xi)(Y - \bar{\omega}V)]^{\frac{1-\xi}{\sigma^*}} \\ &= \left[(1-\xi)(A\chi V^\psi h^\alpha - \bar{\omega}V)\right]^{\frac{1-\xi}{\sigma^*}}, \end{aligned}$$

the problem of the government can be rewritten so as to maximize $\mathcal{U}^g(V, h)$ with respect to TW, under the constraints (D-60) and (D-61) that relate V, h and TW as specified below:

$$\max_{TW} \ \mathcal{U}^g = [1-\xi]^{-\frac{1-\xi}{\sigma^*}} \left[A\chi V^\psi h^\alpha - \bar{\omega}V\right]^{\frac{\sigma^*-1+\xi}{\sigma^*}} - \chi V^\psi \sigma_L \frac{h^{1+\eta}}{1+\eta} \quad , \tag{D-59}$$

$$\text{s.t.} \quad \frac{\overline{\omega}}{\chi} V^{1-\psi} = \epsilon \left[\frac{1+\eta-\alpha}{1+\eta} Ah^\alpha - b\right], \tag{D-60}$$

$$\sigma_L h^{1+\eta} \left[(1-\xi)(A\chi V^\psi h^\alpha - \bar{\omega}V)\right]^{\frac{1-\xi}{\sigma^*}} = \alpha A h^\alpha \frac{1}{TW} \tag{D-61}$$

D.2 Solving the Ramsey problem: Details

Detailing the Ramsey problem The objective (D-59) can be rewritten as:

$$\mathcal{U}^g(h, V) = \widetilde{f}(V, h) - N(V)\Gamma(h)$$

with:

$$\begin{aligned} f(V,h) &= A\chi V^\psi h^\alpha - \bar{\omega}V \\ \widetilde{f}(V,h) &= (1-\xi)^{-\frac{1-\xi}{\sigma^*}} [f(V,h)]^{\frac{\sigma^*-1+\xi}{\sigma^*}} \\ N(V) &= \chi V^\psi \\ \Gamma(h) &= \sigma_L \frac{h^{1+\eta}}{1+\eta} \end{aligned}$$

From this, it comes that:

$$\begin{aligned} \mathcal{U}^{g\prime}_h &= \widetilde{f}'_h - N(V)\Gamma'_h \\ \mathcal{U}^{g\prime}_V &= \widetilde{f}'_V - N'_V \Gamma(h) \end{aligned}$$

with:

$$
\begin{aligned}
\Gamma'_h &= \sigma_L h^\eta \\
N'_V &= \chi\psi V^{\psi-1} \\
f'_h &= A\alpha\chi V^\psi h^{\alpha-1}
\end{aligned}
$$

and:

$$
\begin{aligned}
\widetilde{f}'_h &= (1-\xi)^{-\frac{1-\xi}{\sigma^*}} \frac{\sigma^*-1+\xi}{\sigma^*} f(V,h)^{\frac{\sigma^*-1+\xi}{\sigma^*}-1} f'_h \\
&= [(1-\xi)(Y-\bar{\omega}V)]^{-\frac{1-\xi}{\sigma^*}} \frac{\sigma^*-1+\xi}{\sigma^*} f'_h
\end{aligned}
$$

Such that $\mathcal{U}^{g\prime}_h$ finally writes, with $\frac{\sigma^*-1+\xi}{\sigma^*} = \frac{1}{1+t^*}$:

$$
\mathcal{U}^{g\prime}_h = [(1-\xi)(Y-\bar{\omega}V)]^{-\frac{1-\xi}{\sigma^*}} \left(\frac{1}{1+t^*}\right) A\alpha\chi V^\psi h^{\alpha-1} - \chi V^\psi \sigma_L h^\eta \tag{D-62}
$$

As for $\mathcal{U}^{g\prime}_V$, we have that

$$
\mathcal{U}^{g\prime}_V = \widetilde{f}'_V - N'_V \Gamma(h)
$$

with

$$
\begin{aligned}
N'_V &= \chi\psi V^{\psi-1} \\
f'_V &= A\psi\chi V^{\psi-1} h^\alpha - \bar{\omega} \\
\widetilde{f}'_V &= (1-\xi)^{-\frac{1-\xi}{\sigma^*}} \left(\frac{\sigma^*-1+\xi}{\sigma^*}\right) f(V,h)^{\frac{\sigma^*-1+\xi}{\sigma^*}-1} f'_V \\
&= [(1-\xi)(Y-\bar{\omega}V)]^{-\frac{1-\xi}{\sigma^*}} \frac{\sigma^*-1+\xi}{\sigma^*} f'_V
\end{aligned}
$$

Such that we get for $\mathcal{U}^{g\prime}_V$:

$$
\mathcal{U}^{g\prime}_V = \frac{\psi}{V} \left([(1-\xi)(Y-\bar{\omega}V)]^{-\frac{1-\xi}{\sigma^*}} \left(\frac{1}{1+t^*}\right) \left(\chi A V^\psi h^\alpha - \bar{\omega}\frac{V}{\psi}\right) - \sigma_L \frac{h^{1+\eta}}{1+\eta} \chi V^\psi \right) \tag{D-63}
$$

D.2.1 Difference in marginal utilities: Details

In this section, we investigate the differences in marginal utilities of hours and vacancies between the Ramsey problem and the planners' case. To do so, recall the FOC conditions with respect to vacancies (V) and hours worked (h) respectively as given by Equations (B-48) and (B-47):

$$
\mathcal{U}^{sp\prime}_h = 0 \quad \Leftrightarrow \quad \alpha A \frac{C_H^{\xi-1} C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} - \sigma_L h^{1+\eta-\alpha} = 0 \tag{D-64}
$$

$$
\begin{aligned}
\mathcal{U}^{sp\prime}_V = 0 \quad &\Leftrightarrow \quad \left[\psi A\chi V^{\psi-1} h^\alpha - \bar{\omega}\right] \frac{C_H^{\xi-1} C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} - \sigma_L \frac{h^{1+\eta}}{1+\eta} \chi\psi V^{\psi-1} = 0 \\
&\Leftrightarrow \quad \frac{\psi}{V} \left(\left[A\chi V^\psi h^\alpha - \frac{\bar{\omega}V}{\psi}\right] \frac{C_H^{\xi-1} C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} - \sigma_L \frac{h^{1+\eta}}{1+\eta} \chi V^\psi \right) = 0 \qquad \text{(D-65)}
\end{aligned}
$$

With $C_F = \phi^{\sigma^*-1}$ and C_H and ϕ coming from the planner's program as given by Equations (B-49) and (B-52), with $\mu^* = \frac{\sigma^*}{\sigma^*-1}$ and $1+t^* = \frac{1}{\xi+\frac{1-\xi}{\mu^*}}$. Making use of this, we can rewrite that:

$$\begin{aligned}\frac{C_H^{\xi-1}C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} &= \frac{\phi^{(\sigma^*-1)(1-\xi)}\phi^{\sigma^*(\xi-1)}\left[\frac{\xi}{1-\xi}\mu^*\right]^{\xi-1}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} \\ &= (\mu^*)^{-(1-\xi)}\,\phi^{-(1-\xi)} \\ &= (\mu^*)^{-(1-\xi)}\left[\frac{1+t^*}{\mu^*}\right]^{\frac{-(1-\xi)}{\sigma^*}}\left[(1-\xi)(Y-\bar{\omega}V)\right]^{\frac{-(1-\xi)}{\sigma^*}} \\ &= (\mu^*)^{(1-\xi)(\frac{1}{\sigma^*}-1)}\,(1+t^*)^{\frac{-(1-\xi)}{\sigma^*}}\left[(1-\xi)(Y-\bar{\omega}V)\right]^{\frac{-(1-\xi)}{\sigma^*}}\end{aligned}$$

such that:

$$\frac{C_H^{\xi-1}C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}} = (\mu^*)^{-\frac{1-\xi}{\mu^*}}\,(1+t^*)^{-\frac{1-\xi}{\sigma^*}}\left[(1-\xi)(Y-\bar{\omega}V)\right]^{\frac{-(1-\xi)}{\sigma^*}} \tag{D-66}$$

Replacing this in Equation (D-65), we get that the FOC on vacancies at the planner's solution is given by $\mathcal{U}_V^{sp\prime} = 0$ with $\mathcal{U}_V^{sp\prime}$ equal to:

$$\mathcal{U}_V^{sp\prime} = \frac{\psi}{V}\left\{\left[(1-\xi)(Y-\bar{\omega}V)\right]^{\frac{-(1-\xi)}{\sigma^*}}(\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}}\left[A\chi V^{\psi}h^{\alpha} - \frac{\bar{\omega}V}{\psi}\right] - \sigma_L\frac{h^{1+\eta}}{1+\eta}\chi V^{\psi}\right\} = 0 \tag{D-67}$$

Consider now the planner's FOC on hours worked given by Equation (D-64). Using a similar reasoning as above, we can replace $\frac{C_H^{\xi-1}C_F^{1-\xi}}{\xi^{\xi-1}(1-\xi)^{1-\xi}}$ though Equation (D-66) to obtain that the planner's FOC with respect to hours worked finally writes as:

$$\mathcal{U}_h^{sp\prime} = 0 \;\;\Leftrightarrow\;\; \alpha A h^{\alpha-1}\left[(1-\xi)(Y-\bar{\omega}V)\right]^{-\frac{1-\xi}{\sigma^*}}(\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}} - \sigma_L h^{\eta} = 0$$

Multiplying each term by χV^{ψ}:

$$\alpha A\chi V^{\psi}h^{\alpha-1}\left[(1-\xi)(Y-\bar{\omega}V)\right]^{-\frac{1-\xi}{\sigma^*}}(\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}} - \sigma_L h^{\eta}\chi V^{\psi} = 0 \tag{D-68}$$

As straightforward from the comparison of $\mathcal{U}_V^{g\prime}$ and $\mathcal{U}_V^{sp\prime}$ (Equations (D-63) vs (D-67)), everything else equal for given values of hours worked and vacancies, the marginal utilities of the planner and the government differ as long as:

$$\frac{1}{1+t^*} \neq (\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}} \tag{D-69}$$

The same discrepancy appears when comparing the marginal utilities of hours worked (see Equations (D-62) and (D-68). Making use of the definitions of μ^* and $1+t^*$, such that $\mu^* = \frac{\sigma^*}{\sigma^*-1}$

and $1+t^* = \frac{1}{\xi+\frac{1-\xi}{\mu^*}} = \frac{\sigma^*}{\sigma^*-1+\xi}$, we can rewrite the right-hand side term of Equation (D-69) according to

$$\begin{aligned}
(\mu^*)^{-\frac{1-\xi}{\mu^*}}(1+t^*)^{-\frac{1-\xi}{\sigma^*}} &= (\mu^*)^{-\frac{(1-\xi)(\sigma^*-1}{\sigma^*}}\left[\frac{\mu^*}{\xi\mu^*+1-\xi}\right]^{-\frac{1-\xi}{\sigma^*}} \\
&= (\mu^*)^{\xi-1}\left[\frac{1}{\xi\mu^*+1-\xi}\right]^{-\frac{1-\xi}{\sigma^*}} \\
&= (\frac{\sigma^*}{\sigma^*-1})^{\xi-1}\left[\frac{\sigma^*-1}{\xi\sigma^*+(\sigma^*-1)(1-\xi)}\right]^{-\frac{1-\xi}{\sigma^*}} \\
&= (\sigma^*-1)^{-(1-\xi)(\frac{1}{\sigma^*}-1)}(\sigma^*)^{-(1-\xi)}\left[\sigma^*+\xi-1\right]^{\frac{1-\xi}{\sigma^*}} \\
&= (\sigma^*-1)^{-(1-\xi)(\frac{1}{\sigma^*}-1)}(\sigma^*)^{-(1-\xi)}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{\frac{1-\xi}{\sigma^*}}(\sigma^*)^{\frac{1-\xi}{\sigma^*}} \\
&= \left[\frac{\sigma^*}{\sigma^*-1}\right]^{(1-\xi)(\frac{1}{\sigma^*}-1)}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{\frac{1-\xi}{\sigma^*}} \\
&= \frac{\sigma^*+\xi-1}{\sigma^*}\left[\frac{\sigma^*}{\sigma^*-1}\right]^{(1-\xi)(\frac{1}{\sigma^*}-1)}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{\frac{1-\xi}{\sigma^*}-1} \\
&= \left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)[\mu^*]^{(1-\xi)(\frac{1-\sigma^*}{\sigma^*})}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}} \\
&= \left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)[\mu^*]^{-\frac{1-\xi}{\mu^*}}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}}
\end{aligned}$$

Coming back to Equation (D-69), it exists a gap between the planner's and the Ramsey problem with respect to marginal utilities of labor margins as long as:

$$\frac{\sigma^*-1+\xi}{\sigma^*} \neq \left(\frac{\sigma^*-1+\xi}{\sigma^*}\right)\underbrace{(\mu^*)^{-\frac{1-\xi}{\mu^*}}\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)^{-\frac{\sigma^*+\xi-1}{\sigma^*}}}_{\kappa}$$

that is, as long as $\kappa(\sigma^*) \neq 1$, i.e. as long as

$$\kappa(\sigma^*) \equiv \left(\frac{\sigma^*}{\sigma^*-1}\right)^{-\frac{(1-\xi)(\sigma^*-1)}{\sigma^*}}\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)^{-\frac{\sigma^*+\xi-1}{\sigma^*}} \neq 1 \tag{D-70}$$

Let us focus on the term at the root of the discrepancy between the planner's and the government's problems defined as:

$$\begin{aligned}
\kappa(\sigma^*) &= [\mu^*]^{-\frac{1-\xi}{\mu^*}}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}} \\
&= \left[\frac{\sigma^*}{\sigma^*-1}\right]^{-\frac{(1-\xi)(\sigma^*-1}{\sigma^*}}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}}
\end{aligned}$$

Precisely, decompose $\kappa(\sigma^*)$ in

$$\begin{aligned} \kappa(\sigma^*) &= f(\sigma^*)g(\sigma^*) \\ \text{with} \qquad f(\sigma^*) &= \left[\frac{\sigma^*}{\sigma^*-1}\right]^{-\frac{(1-\xi)(\sigma^*-1}{\sigma^*}} \\ \text{and} \qquad g(\sigma^*) &= \left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}} \end{aligned}$$

First, we want to know how this varies with σ^*. To do so, it is more convenient to consider the logarithm of $\kappa(\sigma^*)$, denoted $\widetilde{\kappa}(\sigma^*) \equiv \ln \kappa(\sigma^*)$:

$$\begin{aligned} \widetilde{\kappa}(\sigma^*) &= \ln\left(\underbrace{\left[\frac{\sigma^*}{\sigma^*-1}\right]^{-\frac{(1-\xi)(\sigma^*-1}{\sigma^*}}}_{f(\sigma^*)}\underbrace{\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}}}_{g(\sigma^*)}\right) \\ &= \underbrace{-\frac{(1-\xi)(\sigma^*-1}{\sigma^*}\ln\left(\frac{\sigma^*}{\sigma^*-1}\right)}_{\widetilde{f}(\sigma^*)}\underbrace{-\frac{\sigma^*+\xi-1}{\sigma^*}\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)}_{\widetilde{g}(\sigma^*)} \\ \rightarrow \quad \widetilde{\kappa}(\sigma^*) &= \widetilde{f}(\sigma^*)+\widetilde{g}(\sigma^*) \end{aligned}$$

with

$$\widetilde{f}(\sigma^*) \equiv \ln f(\sigma^*), \qquad \widetilde{g}(\sigma^*) \equiv \ln g(\sigma^*)$$

As a result, the derivative of $\widetilde{\kappa}(\sigma^*)$ with respect to σ^* can be obtained through

$$\widetilde{\kappa}'(\sigma^*) = \widetilde{f}'(\sigma^*) + \widetilde{g}'(\sigma^*) \tag{D-71}$$

First, we want to calculate $\widetilde{f}'(\sigma^*)$, starting from:

$$\begin{aligned} \widetilde{f}(\sigma^*) &= -\frac{(1-\xi)(\sigma^*-1}{\sigma^*}\ln\left(\frac{\sigma^*}{\sigma^*-1}\right) \\ &= -\frac{(1-\xi)(\sigma^*-1}{\sigma^*}\left[\ln\sigma^* - \ln(\sigma^*-1)\right] \end{aligned}$$

Such that

$$\begin{aligned} \widetilde{f}'(\sigma^*) &= -\frac{(1-\xi)(\sigma^*-1)}{\sigma^*}\left[\frac{1}{\sigma^*}-\frac{1}{\sigma^*-1}\right] - \ln\left(\frac{\sigma^*}{\sigma^*-1}\right)\left[\frac{(1-\xi)\sigma^*-(1-\xi)(\sigma^*-1)}{(\sigma^*)^2}\right] \\ &= -\frac{1}{(\sigma^*)^2}(1-\xi)(\sigma^*-1)\left[1-\frac{\sigma^*}{\sigma^*-1}\right] - \frac{1}{(\sigma^*)^2}(1-\xi)\ln\left(\frac{\sigma^*}{\sigma^*-1}\right) \\ &= \frac{-(1-\xi)}{(\sigma^*)^2}\left[\ln\left(\frac{\sigma^*}{\sigma^*-1}\right)-1\right] \end{aligned}$$

Second, we want to calculate $\widetilde{g}'(\sigma^*)$, starting from:

$$\begin{aligned}
\widetilde{g}'(\sigma^*) &= -\frac{\sigma^*+\xi-1}{\sigma^*}\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right) \\
&= -\frac{\sigma^*+\xi-1}{\sigma^*}\left(\ln(\sigma^*+\xi-1)-\ln(\sigma^*)\right)
\end{aligned}$$

Such that:

$$\begin{aligned}
\widetilde{g}'(\sigma^*) &= -\frac{\sigma^*+\xi-1}{\sigma^*}\left[\frac{1}{\sigma^*+\xi-1}-\frac{1}{\sigma^*}\right]-\ln(\frac{\sigma^*+\xi-1}{\sigma^*})\left(\frac{-\xi+1}{(\sigma^*)^2}\right) \\
&= -\frac{\sigma^*+\xi-1}{(\sigma^*)^2}\left[\frac{\sigma^*}{\sigma^*+\xi-1}-1\right]-\left(\frac{1-\xi}{(\sigma^*)^2}\ln(\frac{\sigma^*+\xi-1}{\sigma^*}\right) \\
&= \frac{-(1-\xi)}{(\sigma^*)^2}\left[1+\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)\right]
\end{aligned}$$

Integrating this into Equation (D-71), we get that

$$\begin{aligned}
\widetilde{\kappa}'(\sigma^*) &= \widetilde{f}'(\sigma^*)+\widetilde{g}'(\sigma^*) \\
&= \frac{-(1-\xi)}{(\sigma^*)^2}\left[\ln\left(\frac{\sigma^*}{\sigma^*-1}\right)-1\right]-\frac{1-\xi}{(\sigma^*)^2}\left[1+\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)\right] \\
&= \frac{-(1-\xi)}{(\sigma^*)^2}\left[\ln\left(\frac{\sigma^*}{\sigma^*-1}\right)+\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)\right] \\
&= \frac{-(1-\xi)}{(\sigma^*)^2}\left[\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*-1}\right)\right]
\end{aligned}$$

Given that $\xi > 0$, then $\ln\left(\frac{\sigma^*+\xi-1}{\sigma^*-1}\right) > 1$, implying $\widetilde{\kappa}'(\sigma^*) < 0$.

This means that the discrepancy term $\kappa \equiv [\mu^*]^{-\frac{1-\xi}{\mu^*}}\left[\frac{\sigma^*+\xi-1}{\sigma^*}\right]^{-\frac{\sigma^*+\xi-1}{\sigma^*}}$ is decreasing with σ^*.

Consider Equation (D-70). We now determine the limit values of $\kappa(\sigma^*)$ that scales the discrepancy in marginal utilities of V.

When $\sigma^* \to \infty$, then $\mu^* \to 1$. Further,

$$\lim_{\sigma^*\to\infty}\frac{\sigma^*+\xi-1}{\sigma^*} = \lim_{\sigma^*\to\infty} 1+\frac{\xi-1}{\sigma^*} = 1$$

Hence, it comes that:

$$\begin{aligned}
\text{Under} \quad \sigma^*\to\infty, \qquad & (\mu^*)^{-\frac{1-\xi}{\mu^*}}\left(\frac{\sigma^*+\xi-1}{\sigma^*}\right)^{-\frac{\sigma^*+\xi-1}{\sigma^*}} = 1 \\
\Leftrightarrow \qquad & \mathcal{U}_V^{g\prime} = \mathcal{U}_V^{sp\prime}, \quad \text{and} \quad \mathcal{U}_h^{g\prime} = \mathcal{U}_h^{sp\prime}
\end{aligned}$$

Consider now the limit value of κ when $\sigma^* \to 1$. To determine this value, it is convenient to rewrite Equation (D-70) as:

$$\kappa = [\sigma^* - 1]^{\frac{(1-\xi)(\sigma^*-1)}{\sigma^*}} (\sigma^*)^{\xi} \left(\frac{1}{\sigma^* + \xi - 1} \right)^{\frac{\sigma^*+\xi-1}{\xi}}$$

With

$$\lim_{\sigma^* \to 1} [\sigma^* - 1]^{\frac{(1-\xi)(\sigma^*-1)}{\sigma^*}} = 1$$

and

$$\lim_{\sigma^* \to 1} \left(\frac{1}{\sigma^* + \xi - 1} \right)^{\frac{\sigma^*+\xi-1}{\xi}} = \xi^{-\xi}$$

we get that $\lim_{\sigma^* \to 1} \kappa = \xi^{-\xi} > 1$. Summarizing our results:

$$\text{When } \sigma^* \to 1, \quad \lim_{\sigma^* \to 1} \kappa = \xi^{-\xi} > 1 \quad \to \quad \mathcal{U}_V^{g\prime} < \mathcal{U}_V^{sp\prime}, \quad \text{and} \quad \mathcal{U}_h^{g\prime} < \mathcal{U}_h^{sp\prime}$$
$$\text{When } \sigma^* \to \infty, \quad \lim_{\sigma^* \to \infty} \kappa = 1, \quad \to \quad \mathcal{U}_V^{g\prime} = \mathcal{U}_V^{sp\prime}, \quad \text{and} \quad \mathcal{U}_h^{g\prime} = \mathcal{U}_h^{sp\prime}$$