Technical Appendix on Financial Frictions

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Abstract

The above four models are compared from the point of view of their implications for the dynamic response of the economy to a technology shock and to a monetary policy shock. There are four results. First, it makes essentially no difference whether markets are complete or not. Second, a parameterization is found having the following two implications: (i) in the wake of a technology shock, the model without financial frictions (CEE) responds in roughly the same way as the model with financial frictions. This is explained as reflecting the fact that Irving Fisher's 'Fisher price deflation effect' cancels with income and capital gains effects; (ii) in the wake of a monetary policy shock, the model with financial frictions responds much more strongly than does CEE because in this case the Fisher price effects and the income and capital gains effects work in the same direction. Our third result is that, while (i) and (ii) reflect a plausible parameterization, alternative (seemingly) plausible parameterizations produce models with properties different than (i) and (ii). Fourth, without additional frictions the model with just financial frictions appears to have counterfactual implications for what happens after a monetary policy shock.

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1. CEE Model

1.1. Model

Final good firm technology:

$$Y_t = \left[\int_0^1 Y_{jt}{}^{\frac{1}{\lambda_t}} dj \right]^{\lambda_f}, \ 1 \leq \lambda_f < \infty, \tag{1.1}$$

Intermediate good j is produced by a price-setting monopolist with technology:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha} (z_t l_{jt})^{1-\alpha} - \Phi z_t & \text{if } \epsilon_t K_{jt}^{\alpha} (z_t l_{jt})^{1-\alpha} > \Phi z_t \\ 0, & \text{otherwise} \end{cases}, \ 0 < \alpha < 1, \tag{1.2}$$

where Φz_t is a fixed cost and K_{jt} and l_{jt} denote the services of capital and homogeneous labor. Capital and labor services are hired in competitive markets at nominal prices, $P_t r_t^k$, and W_t, respectively. The object, z_t, in (1.2), is assumed to evolve deterministically:

$$z_t = z_{t-1} \mu_z. \tag{1.3}$$

Law of motion of technology, ϵ_t :

$$\log(\epsilon_t) = \rho \log(\epsilon_{t-1}) + \varepsilon_t^{\epsilon}.$$

We adopt a variant of Calvo sticky prices. In each period, t, a fraction of intermediate-goods firms, $1 - \xi_p$, can reoptimize their price. If the i^{th} firm in period t cannot reoptimize, then it sets price according to:

$$P_{it} = \tilde{\pi}_t P_{i,t-1},$$

where

$$\tilde{\pi}_t = \pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}. \tag{1.4}$$

Here, π_t denotes the gross rate of inflation, $\pi_t = P_t / P_{t-1}$, and $\bar{\pi}$ denotes a constant (in practice, it would be set to unity or to steady state inflation). If the i^{th} firm is permitted to optimize its price at time t, it chooses $P_{i,t} = \tilde{P}_t$ to optimize discounted profits:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \lambda_{t+j} \left[P_{i,t+j} Y_{i,t+j} - P_{t+j} s_{t+j} (Y_{i,t+j} + \Phi z_{t+j}) \right]. \tag{1.5}$$

Here, λ_{t+j} is the multiplier on firm profits in the household's budget constraint. Also, $P_{i,t+j}$, $j > 0$ denotes the price of a firm that sets $P_{i,t} = \tilde{P}_t$ and does not reoptimize between $t+1, ..., t+j$. The equilibrium conditions associated with firms appear in the next subsection.

The household maximizes utility

$$E_t^j \sum_{l=0}^{\infty} \beta^{l-t} \left\{ u(c_{t+l}) - \psi_L \frac{h_{j,t}^{1+\sigma_L}}{1+\sigma_L} - \upsilon \frac{\left(\frac{P_{t+l} c_{t+l}}{M_{t+l}^d} \right)^{1-\sigma_q}}{1-\sigma_q} \right\}, \ \upsilon \simeq 0, \tag{1.6}$$

subject to the constraint

$$P_t\left(c_t+i_t\right)+M_{t+1}^d-M_t^d+T_{t+1}\leq W_{t,j}l_{t,j}+P_t r_t^k k_t+\left(1+R_t^e\right)T_t, \tag{1.7}$$

where M_t^d denotes the household's beginning-of-period stock of money and T_t denotes nominal bonds issued in period $t-1$, which earn interest, R_t^e, in period t. This nominal interest rate is known at $t-1$. The household's problem is to maximize (1.6) subject to the capital accumulation technology linking investment, i, to capital.

The j^{th} household, $j\in(0,1)$, supplies a differentiated labor service which is aggregated into a homogeneous labor good by perfectly competitive labor contractors using the following constant returns to scale technology:

$$l_t=\left[\int_0^1\left(h_{t,j}\right)^{\frac{1}{\lambda_w}}dj\right]^{\lambda_w},\ 1\leq\lambda_w<\infty. \tag{1.8}$$

Aggregate labor is sold competitively by the representative labor contractor to intermediate goods producers for wage W_t and the j^{th} household's wage is $W_{j,t}$. The contractor hires $h_{t,j}$, $j\in(0,1)$, in order to maximize profits:

$$\max_{h_{t,j}} W_t\left[\int_0^1\left(h_{t,j}\right)^{\frac{1}{\lambda_w}}dj\right]^{\lambda_w}-\int_0^1 W_{t,j}h_{t,j}dj,$$

which leads to the first order condition:

$$W_t l_t^{\frac{\lambda_w-1}{\lambda_w}}\left(h_{t,j}\right)^{\frac{1-\lambda_w}{\lambda_w}}=W_{t,j},$$

or,

$$h_{t,j}=l_t\left[\frac{W_{t,j}}{W_t}\right]^{\frac{\lambda_w}{1-\lambda_w}}. \tag{1.9}$$

The j^{th} household views (1.9) as a demand curve for its specialized labor services. The rules are that if the household posts a wage, $W_{t,j}$, then it must supply the services, $h_{t,j}$, implied by the demand curve.

Thus, the household's problem is to choose its wage rate, $W_{t,j}$. With probability, $1-\xi_w$, it can optimize its wage rate and with the complementary probability, it cannot. In this case, we suppose that it sets its wage as follows:

$$W_{t,j}=\tilde{\pi}_{w,t-1}W_{t-1,j},\ \tilde{\pi}_{w,t}\equiv\left(\pi_{t-1}\right)^{\iota_w}\bar{\pi}^{1-\iota_w}. \tag{1.10}$$

The $1-\xi_w$ households that set their wage optimally in period t all find it optimal to set the same wage, $\tilde{W}_t$. The household which can optimize its wage in period t does so to optimize the following objective:

$$E_t\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i\{-\frac{\psi_L}{1+\sigma_L}h_{j,t+i}^{1+\sigma_L}+\lambda_{t+i}W_{j,t+i}h_{j,t+i}\},$$

where λ_{t+i} is the multiplier on the household's budget constraint, (1.7). The household discounts by $\beta\xi_w$ because it is only interested in continuation histories in which it does not reoptimize its period t wage.

1.2. Equilibrium Conditions

The equations pertaining to prices are:

$$(1)p_t^* - \left[(1-\xi_p) \left(\frac{1-\xi_p \left(\frac{\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right)^{\lambda_f} + \xi_p \left(\frac{\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right]^{\frac{1-\lambda_f}{\lambda_f}} = 0 \quad (1.11)$$

and

$$(2)E_t \left\{ \lambda_{z,t} \left(p_t^*\right)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_z} \right)^{\alpha} \left(\left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t \right)^{1-\alpha} - \phi \right] + \left(\frac{\pi_t^{\iota}\bar{\pi}^{1-\iota}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0, \quad (1.12)$$

where $\lambda_{z,t}$ denotes $\lambda_t z_t P_t$. Also,

$$(3)\lambda_{z,t}\lambda_f \left(p_t^*\right)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\epsilon_t \left(\frac{k_t}{\mu_z} \right)^{\alpha} \left(\left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t \right)^{1-\alpha} - \phi \right] s_t + \quad (1.13)$$

$$\beta \xi_p \left(\frac{\pi_t^{\iota_2}\bar{\pi}^{1-\iota_2}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \left[\frac{1-\xi_p \left(\frac{\pi_t^{\iota}\bar{\pi}^{1-\iota}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right]^{1-\lambda_f} F_{p,t+1} - F_{p,t} \left[\frac{1-\xi_p \left(\frac{\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right]^{1-\lambda_f} = 0.$$

Note that both these equations involve $F_{p,t}$. This reflects that a lot of equations have been substituted out. In particular, we have

$$F_{p,t} \left[\frac{1-\xi_p \left(\frac{\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right]^{1-\lambda_f} = K_{p,t}, \ \tilde{p}_t = \frac{K_{p,t}}{F_{p,t}},$$

where $\tilde{p}_t$ is the price set by price-optimizing firms in period t. In addition, $\tilde{p}_t$ is substituted out using the equilibrium condition relating the aggregate price level to the prices of intermediate goods.

Now, consider the wage equations. The wages of non-optimizing households evolve as follows:

$$W_{j,t} = \tilde{\pi}_{w,t}\mu_z W_{j,t-1}, \ \tilde{\pi}_{w,t} \equiv \left(\pi_{t-1}\right)^{\iota_w} \bar{\pi}^{1-\iota_w}. \quad (1.14)$$

Nominal wage growth, $\pi_{w,t}$, is:

$$\pi_{w,t} = \frac{\tilde{w}_t \mu_z \pi_t}{\tilde{w}_{t-1}},$$

where

$$\tilde{w}_t \equiv \frac{W_t}{z_t P_t}.$$

The optimality conditions associate with wage-setting are characterized by:

$$(4) E_t\{\lambda_{z,t}\frac{(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t}{\lambda_w} + \beta\xi_w \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}} \frac{\left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}\pi_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t}\} = 0 \tag{1.15}$$

and

$$(5)\ E_t\{\left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t\right]^{1+\sigma_L} \tag{1.16}$$

$$+\beta\xi_w\left(\frac{\tilde{\pi}_{w,t+1}}{\tilde{w}_{t+1}\pi_{t+1}}\tilde{w}_t\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \frac{1}{\psi_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t+1}}{\tilde{w}_{t+1}\pi_{t+1}}\tilde{w}_t\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \tilde{w}_{t+1}F_{w,t+1}$$

$$-\frac{1}{\psi_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\tilde{w}_t\pi_t}\tilde{w}_{t-1}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \tilde{w}_t F_{w,t}\} = 0.$$

The law of motion of the labor market distortion is:

$$(6) \qquad w_t^* = [(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\tilde{w}_t\pi_t}\tilde{w}_{t-1}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\tilde{w}_t\pi_t}\tilde{w}_{t-1}w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{1.17}$$

Marginal cost:

$$(7) s_t = \frac{\tilde{w}_t}{(1-\alpha)\epsilon_t}\left(\frac{\mu_z \overbrace{(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} \overbrace{h_t}^{\text{number of people}}}^{\text{total effective labor, taking into account wage distortions}}}{k_t}\right)^{\alpha} \tag{1.18}$$

Resource constraint:

$$(8) c_t + I_t = (p_t^*)^{\frac{\lambda_f}{\lambda_f-1}}\left\{\epsilon_t\left(\frac{k_t}{\mu_z}\right)^{\alpha}\left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}} h_t\right]^{1-\alpha} - \phi\right\} \tag{1.19}$$

where

$$(9) k_{t+1} - (1-\delta)\mu_z^{-1}k_t = \left[1 - \frac{S''\mu_z^2}{2}(\frac{I_t}{I_{t-1}} - 1)^2\right] I_t, \tag{1.20}$$

where I_t is investment scaled by z_t.

Equation defining the nominal non-state contingent rate of interest:

$$(10) E_t\{\beta\frac{1}{\pi_{t+1}\mu_z}\lambda_{z,t+1}(1+R_t) - \lambda_{z,t}\} = 0 \tag{1.21}$$

The derivative of utility with respect to consumption (after scaling by multiplying by z_t),

$$(11) E_t \left[\lambda_{z,t} - \frac{\mu_z}{c_t \mu_z - b c_{t-1}} + b\beta \frac{1}{\mu_z c_{t+1} - b c_t} \right] = 0, \tag{1.22}$$

where c_t denotes consumption scaled by z_t. The capital first order condition:

$$(12) - \lambda_{z,t} + E_t \lambda_{z,t+1} \frac{\beta}{\mu_z} \frac{1 + R_{t+1}^k}{\pi_{t+1}} = 0, \tag{1.23}$$

where R_{t+1}^k denotes the rate of return on capital:

$$(13) 1 + R_t^k = \frac{r_t^k + (1-\delta) q_t}{q_{t-1}} \pi_t, \; r_t^k = \alpha \epsilon_t \left(\frac{\mu_z \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t}{k_t} \right)^{1-\alpha} s_t,$$

where q_t denotes the market price of capital, k_{t+1}, scaled by the price, P_t, of homogeneous goods. The investment first order condition:

$$\begin{aligned} (14) E_t \{ \lambda_{zt} q_t & \left[1 - \frac{S'' \mu_z^2}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - S'' \mu_z^2 \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ & - \lambda_{zt} + \beta \lambda_{zt+1} q_{t+1} S'' \mu_z^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \} = 0, \end{aligned} \tag{1.24}$$

where I_t is interpreted as investment, divided by z_t.

The monetary policy rule:

$$\begin{aligned} (15) \; \log(1 + R_t) &= (1 - \tilde{\rho}) \log(1 + R) + \tilde{\rho} \log(1 + R_{t-1}) \\ & + \frac{1 - \tilde{\rho}}{1 + R} \left[\tilde{a}_p \pi \log \frac{\pi_{t+1}}{\pi} + \tilde{a}_y \frac{1}{4} \log \frac{y_t}{y} \right] + x_t^p, \end{aligned} \tag{1.25}$$

where x_t^p is an iid monetary policy shock and y_t denotes output, $c_t + I_t$.

There are 15 equilibrium conditions in 15 unknowns:

$$\lambda_{zt}, \; h_t, \; c_t, \; I_t, \; y_t, \; q_t, \; \pi_t, \; \tilde{w}_t, \; w_t^*, \; p_t^*, \; F_{p,t}, \; k_t, \; s_t, \; R_t, \; R_t^k.$$

1.3. The Flexible Price Case

It is interesting to consider the special case of price flexibility, i.e., when $\xi_p = \xi_w = 0$. In this case, (1) implies $p_t^* = 1$, (2)(3) imply $s_t = 1/\lambda_f$, (6) implies $w_t^* = 1$, and the remaining equations imply:

$$\frac{1}{\lambda_f} = \frac{\lambda_w \frac{\psi_L h_t^{\sigma_L}}{\lambda_{z,t}}}{(1-\alpha)\,\epsilon_t}\left(\frac{\mu_z h_t}{k_t}\right)^{\alpha} \tag{1.26}$$

$$c_t + I_t = \epsilon_t \left(\frac{k_t}{\mu_z}\right)^{\alpha} h_t^{1-\alpha} - \phi \tag{1.27}$$

$$k_{t+1} - (1-\delta)\mu_z^{-1} k_t = \left[1 - \frac{S''\mu_z^2}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t \tag{1.28}$$

$$E_t\{\beta \frac{1}{\mu_z}\lambda_{z,t+1}\frac{1+R_t}{\pi_{t+1}} - \lambda_{z,t}\} = 0 \tag{1.29}$$

$$E_t\left[\lambda_{z,t} - \frac{\mu_z}{c_t\mu_z - bc_{t-1}} + b\beta\frac{1}{\mu_z c_{t+1} - bc_t}\right] = 0 \tag{1.30}$$

$$\begin{aligned} E_t\{\lambda_{zt} q_t \left[1 - \frac{S''\mu_z^2}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - S''\mu_z^2\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] & \\ -\lambda_{zt} + \beta\lambda_{zt+1} q_{t+1} S''\mu_z^2\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\} &= 0 \end{aligned} \tag{1.31}$$

and the household's intertemporal equation:

$$-\lambda_{z,t} + E_t\lambda_{z,t+1}\frac{\beta}{\mu_z}\frac{\alpha\epsilon_t\left(\frac{\mu_z h_t}{k_t}\right)^{1-\alpha}\frac{1}{\lambda_f} + (1-\delta)q_t}{q_{t-1}} = 0 \tag{1.32}$$

This represents 7 equations in 7 unknowns: k_{t+1}, c_t, I_t, h_t, $\lambda_{z,t}$, q_t, $(1+R_t)/\pi_{t+1}$. Another equation is required to disentangle inflation and R_t from the real rate. Consider the possibility that the Taylor rule could do this:

$$\begin{aligned} (15)\ \log(1+R_t) &= (1-\tilde{\rho})\log(1+R) + \tilde{\rho}\log(1+R_{t-1}) \\ &\quad + \frac{1-\tilde{\rho}}{1+R} E_t\left[\tilde{a}_p \pi \log\frac{\pi_{t+1}}{\pi} + \tilde{a}_y \frac{1}{4}\log\frac{y_t}{y}\right] + x_t^p, \end{aligned}$$

Note that this equation cannot be solved for π_t conditional on the real rate and other variables, because of the presence of π_{t+1}. When this is replaced by current inflation, however, the system does have a unique solution.

2. Adding Financial Frictions to CEE

We now add a version of the financial frictions proposed by Bernanke, Gertler and Gilchrist. This introduces 3 new relations: the optimality condition associated with the standard debt contract offered to entrepreneurs, a zero profit condition on banks and the law of motion for entrepreneurial net worth.

2.1. The Additional Equilibrium Conditions

A group of households, 'entrepreneurs', own the capital stock and rent it out. The sequence of events from the close of goods markets in period t to close of the goods markets in period $t+1$ is as follows:

- At close of period t goods markets, entrepreneurs' state is their net worth, and they go to a bank where they receive a \loan contract.
- Their loan is combined with their net worth to buy the entire stock of capital, k_{t+1}, that can be rented in period $t+1$.
- Each entrepreneur receives an idiosyncratic shock, so that their capital becomes ωk_{t+1}.
- In period $t+1$ the entrepreneur rents the capital in a competitive market for capital services.
- In period $t+1$ the entrepreneur sells the entire stock of capital and the period t loan contract is brought to an end. The entrepreneur's period $t+1$ net worth is determined.
- A fraction, γ_{t+1}, of entrepreneurs dies (the entrepreneurs who die receive a last meal, the only consumption they ever get). The complementary fraction is born. Surviving and newborn entrepreneurs all receive a (small) lump sum transfer.
- Entrepreneurs go to the bank to receive a new loan contract.

We now describe what happens to entrepreneurs in detail. After the period t goods market closes, entrepreneurs have net worth, n_{t+1}, and they borrow B_{t+1} in order to finance their purchases of the stock of capital:

$$B_{t+1} + n_{t+1} = q_t k_{t+1}.$$

Here, B_{t+1} and n_{t+1} are both divided by $P_t z_t$. (Recall, q_t is the period t currency price of capital, scaled by P_t, and k_{t+1} is the end of period t stock of capital, scaled by z_t.) After purchasing capital, each entrepreneur receives a mean-unity idiosyncratic shock, ω, which has log-normal cdf, F_t. The subscript here reflects that the variance of $\log \omega$, σ_t^2, is itself a random variable which is realized at time t. This random variable controls the riskiness of individual entrepreneurs, and this has aggregate consequences. We refer to σ_t as 'risk'.

Entrepreneurs receive a standard debt contract from banks, which specifies that they pay Z_{t+1} in interest on their bank loans in period $t+1$, if it is feasible for them to do so. Entrepreneurs who draw a low value of ω declare bankruptcy, are monitored, and have everything taken away from them by the bank. The cutoff value of ω which divides bankrupt from non-bankrupt entrepreneurs, satisfies,

$$\bar{\omega}_{t+1} \left(1 + R_{t+1}^k\right) q_t k_{t+1} = Z_{t+1} B_{t+1}. \tag{2.1}$$

In the benchmark version of our model, we follow BGG in supposing that markets are incomplete. In particular, banks must satisfy a state-by-state zero profit condition:

$$\left[1 - F_t\left(\bar{\omega}_{t+1}\right)\right] Z_{t+1} B_{t+1} + (1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) \left(1 + R_{t+1}^k\right) q_t k_{t+1} = (1 + R_t) B_{t+1}. \tag{2.2}$$

From the point of view of households, the assumed absence of complete markets reflects our assumption that the only opportunities for intertemporal trade available to them is their non-state contingent deposits with banks. From the perspective of banks, the only agents with which they could in principle trade in complete markets is households and other banks. Households are ruled out by assumption and other banks are ruled out by the assumption that all banks are identical. An implication of the fact that (2.1) and (2.2) must hold in each state of nature is that $\bar{\omega}_{t+1}$ and Z_{t+1} must both vary across the period $t+1$ state of nature. The fact that interest paid by entrepreneurs varies across states of nature is perhaps implausible, and for this reason we consider an alternative formulation below.[1]

The first term on the left of (2.2) represents the revenues received from non-bankrupt entrepreneurs, who pay gross interest, Z_{t+1}, and borrow B_{t+1} in period t. The next term is the total receipts from entrepreneurs who are bankrupt, net of monitoring costs. Rearranging (2.2) and making use of the cutoff equation we obtain:

$$(16)\ \Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{1+R_t}{1+R_{t+1}^k}\left(1 - \frac{n_{t+1}}{q_t k_{t+1}}\right), \tag{2.3}$$

which must hold in each realized $t+1$ state of nature. Here,

$$\begin{aligned} \overbrace{\Gamma_t(\bar{\omega}_{t+1})}^{\text{share of entrepreneurial earnings, } \left(1+R_{t+1}^k\right)q_t k_{t+1}\text{, received by bank}} &\equiv \bar{\omega}_{t+1}\left[1 - F_t(\bar{\omega}_{t+1})\right] + G_t(\bar{\omega}_{t+1}) \\ G_t(\bar{\omega}_{t+1}) &\equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega). \end{aligned} \tag{2.4}$$

The loan contract must satisfy an optimality condition. To derive this condition, note that entrepreneurs' utility at the termination of the contract entered into at date t is given by

$$\frac{\overbrace{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[\left(1+R_{t+1}^k\right)\omega q_t k_{t+1} - Z_{t+1}B_{t+1}\right] dF_t(\omega)}^{\text{net worth of entrepreneurs at conclusion of date } t \text{ debt contract, scaled by } P_t z_t}}{\underbrace{}_{}\overbrace{\left(1+R_t\right)n_{t+1}}^{\text{opportunity cost of entrepreneurial funds, scaled by } P_t z_t}}$$

$$= E_t\left\{\overbrace{\left[1 - \Gamma_t(\bar{\omega}_{t+1})\right]}^{\text{share of entrepreneurial earnings kept by entrepreneur}} \times \frac{1+R_{t+1}^k}{1+R_t}\right\}\varrho_t,\ \varrho_t \equiv \frac{\overbrace{q_t k_{t+1}}^{\text{assets to net worth ratio}}}{n_{t+1}} \tag{2.5}$$

Here, we have scaled entrepreneurial utility by the opportunity cost of their funds. In equilibrium, the loan contract (which we can parameterize by ϱ_t and $\bar{\omega}_{t+1}$[2]) must optimize

[1] BGG present a defense of the implication that Z_{t+1} varies across states of nature in footnte 10.

[2] To see this, note

$$B_{t+1} = n_{t+1}\left(\varrho_t - 1\right),\ Z_{t+1} = \frac{\varrho_t}{\varrho_t - 1}\left(1+R_{t+1}^k\right)\bar{\omega}_{t+1}.$$

the entrepreneur's utility, subject to the bank's zero profit condition. In Lagrangian form, the problem is:

$$\max_{\varrho_t,\{\bar{\omega}_{t+1}\}} E_t\Big\{\frac{[1-\Gamma_t(\bar{\omega}_{t+1})]\left(1+R^k_{t+1}\right)\varrho_t}{1+R_t} \tag{2.6}$$

$$+\eta_{t+1}\overbrace{\left(\frac{[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\left(1+R^k_{t+1}\right)\varrho_t}{1+R_t}-\varrho_t+1\right)}^{\text{state-by-state bank zero profit condition}}\Big\}.$$

The choice variables in the optimization problem are the single ϱ_t and one $\bar{\omega}_{t+1}$ for each possible period $t+1$ state of nature. The cost of funds and the rate of return on capital are taken as given in this optimization problem because banks are competitive. The first order necessary conditions for optimality are:

$$\begin{aligned}
E_t\left\{[1-\Gamma_t(\bar{\omega}]\frac{1+R^k_{t+1}}{1+R_t}+\eta_{t+1}\left([\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R_t}-1\right)\right\} &= 0\\
-\Gamma_t'(\bar{\omega}_{t+1})+\eta_{t+1}\left[\Gamma_t'(\bar{\omega}_{t+1})-\mu G_t'(\bar{\omega}_{t+1})\right] &= 0\\
[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1 &= 0.
\end{aligned}$$

Substituting out for η_{t+1} from the second first order condition into the first, we obtain:

$$\begin{aligned}
&E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R_t}+\frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1})-\mu G_t'(\bar{\omega}_{t+1})}\left[\frac{1+R^k_{t+1}}{1+R_t}\left(\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})\right)-1\right]\right\}\\
&= 0.
\end{aligned}$$

In principle these equations should have been derived separately for entrepreneurs with each different level of possible net worth. It is clear from the first order conditions that had we done so, each one's standard debt contract would have been characterized by the same ϱ_t, $\{\bar{\omega}_{t+1}\}$. Using the facts, $\Gamma'=1-F$ and $G'=\bar{\omega}F'$, we obtain

$$\begin{aligned}
&(17) E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R_t}+\frac{1-F_t(\bar{\omega}_{t+1})}{1-F_t(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F_t'(\bar{\omega}_{t+1})}\left[\frac{1+R^k_{t+1}}{1+R_t}\left(\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})\right)-\right.\right.\\
&= 0.
\end{aligned}$$

We now derive the law of motion of net worth. After the loan contract received in $t-1$ is settled, but before it is known which entrepreneur exits and which stays, the (scaled by $P_t z_t$) net worth in period t of entpprepreneurs is

$$V_t = \overbrace{[1-\Gamma_{t-1}(\bar{\omega}_t)]}^{\text{share of entrepreneurial earnings received by entrepreneurs}} \times\left(1+R^k_t\right)\frac{q_{t-1}}{\pi_t\mu_z}k_t,$$

where the appearance of $\pi_t\mu_z$ in the denominator reflects that $q_{t-1}k_t$ has been scaled by $P_{t-1}z_{t-1}$. The above expression can be written

$$\begin{aligned}
V_t &= \overbrace{\left\{1-\bar{\omega}_t\left[1-F_{t-1}(\bar{\omega}_t)\right]-\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\right\}}^{=1-\Gamma_{t-1}(\bar{\omega}_t)}\left(1+R_t^k\right)\frac{q_{t-1}}{\pi_t\mu_z}k_t \\
&= \left(1+R_t^k\right)\frac{q_{t-1}}{\pi_t\mu_z}k_t-\overbrace{\left(\bar{\omega}_t\left[1-F_{t-1}(\bar{\omega}_t)\right]+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\right)\left(1+R_t^k\right)\frac{q_{t-1}}{\pi_t\mu_z}k_t}^{\text{earnings of banks, which must equal } B_t(1+R_{t-1})=(1+R_{t-1})(q_{t-1}k_t-n_t)} \\
&\quad -\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\frac{q_{t-1}}{\pi_t\mu_z}k_t \\
&= \left[1+R_t^k-(1+R_{t-1})-\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\right]\frac{q_{t-1}}{\pi_t\mu_z}k_t+\frac{(1+R_{t-1})}{\pi_t\mu_z}n_t.
\end{aligned}$$

At this point, γ_t entrepreneurs exit and are replaced by γ_t new arrivals. Both surviving entrepreneurs and new arrivals receive a lump sum transfer in the amount, w^e. Thus, $n_{t+1}=\gamma_t V_t+w^e$, or,

$$(18)n_{t+1}=\frac{\gamma_t}{\pi_t\mu_z}\left\{R_t^k-R_{t-1}-\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\right\}k_t q_{t-1}+w^e+\gamma_t\left(\frac{1+R_{t-1}}{\pi_t\mu_z}\right)n_t. \tag{2.8}$$

There is a way to rewrite this expression in a way that conveys additional intuition. Recall the identity, $B_t+n_t=q_{t-1}k_t$. Substitute this into the above expression:

$$\begin{aligned}
n_{t+1} &= \frac{\gamma_t}{\pi_t\mu_z}\left\{R_t^k-R_{t-1}-\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\right\}k_t q_{t-1}+w^e+\gamma_t\left(\frac{1+R_{t-1}}{\pi_t\mu_z}\right)(q_{t-1}k_t-B_t) \\
&= \frac{\gamma_t}{\mu_z}\left[1-\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\right]\frac{1+R_t^k}{\pi_t}k_t q_{t-1}+w^e-\gamma_t\left(\frac{1+R_{t-1}}{\pi_t\mu_z}\right)B_t, \qquad (2.9)
\end{aligned}$$

after cancelling. Recall that $1+R_t^k$ is proportional to π_t, so that inflation does not affect the earnings of entrepreneurs. However, a jump in π_t does reduce the payments on debt, and so enhances their net worth in this way.

The resource constraint becomes:

$$d_t+c_t+I_t+\Theta\frac{1-\gamma_t}{\gamma_t}\left[n_{t+1}-w^e\right]=(p_t^*)^{\frac{\lambda_f}{\lambda_f-1}}\left\{\epsilon_t\left(\frac{k_t}{\mu_z^*}\right)^{\alpha}\left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}L_t\right]^{1-\alpha}-\phi\right\} \tag{2.10}$$

Here, $\left[n_{t+1}-w^e\right]/\gamma_t$ denotes the assets of entrepreneurs before they have received their real transfer, w^e, and before it is determined if they are to be selected to exit. The assets of the fraction of entrepreneurs that exit is $(1-\gamma_t)$ times this amount, and they consume Θ of their assets, with the other $1-\Theta$ being transferred to households. Also, d_t denotes the resources used up in monitoring:

$$d_t=\frac{\mu G(\bar{\omega}_t)\left(1+R_t^k\right)q_{t-1}k_t}{\mu_z^*}\frac{1}{\pi_t}.$$

In the modified economy, entrepreneurs rather than households accumulate capital. This means that the household intertemporal equation, (1.23), (i.e., (12)) must be deleted. So, we have added three new equations, (2.7), (2.3) and (2.8) and deleted one. The net increase in the number of equations is two. We increase the number of endogenous variables by two: $\bar{\omega}_{t+1}$ and n_{t+1} (the first variable is a function of the period $t+1$ state of nature, while the second is a function of the period t state of nature).

2.2. Some Additional Technical Details for the Model

This subsection evaluates various integrals and derivates pertaining to the log-normal cdf. The log-normal pdf has two parameters, the variance of $\log \omega$ and the mean of ω. We set the mean to unity and we calibrate the steady state value of the variace so that, in steady state, $F(\bar{\omega})$ is equal to a specified calibrated value. To evaluate (8.9) it is useful to have a formula for computing

$$G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega).$$

Thus, note $\omega = e^x$, $d\omega = e^x dx$, $x = \log \omega$, $dx = d\omega/\omega$

$$\int_0^{\bar{\omega}} \omega dF(\omega) = \int_{-\infty}^{\log \bar{\omega}} e^x f(x)\, dx,$$

where $x = \log(\omega)$ and f is the Normal density function. Writing this explicitly:

$$\begin{aligned}
\int_0^{\bar{\omega}} \omega dF(\omega) &= \int_{-\infty}^{\log \bar{\omega}} e^x f(x)\, dx \\
&= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^x \exp^{\frac{-(x-Ex)^2}{2\sigma_x^2}} dx,
\end{aligned}$$

where σ_x^2 is the variance of x. Now, $E\omega = 1$ implies $Ex = -(1/2)\sigma_x^2$, so that

$$\begin{aligned}
\int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}} e^x \exp^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx \\
&= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}_t} \exp^{\frac{x2\sigma_x^2-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx \\
&= \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{\log \bar{\omega}_t} \exp^{\frac{-\left(x-\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx.
\end{aligned}$$

Now, make the change of variable,

$$\begin{aligned}
v &= \frac{x - \frac{1}{2}\sigma_x^2}{\sigma_x} = \frac{x + \frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x \\
\bar{v} &= \frac{\log(\bar{\omega}) + \frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x \\
dv &= \frac{1}{\sigma_x} dx
\end{aligned}$$

so that

$$\begin{aligned}
\int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{\frac{-v^2}{2}} \sigma_x dv \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{\frac{-v^2}{2}} dv \\
&= prob\left[x < \frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x\right].
\end{aligned}$$

where

$$\begin{aligned}
E\omega &= Ee^x = e^{\left[Ex+\frac{1}{2}\sigma_x^2\right]} = 1 \\
Ex &= -\frac{1}{2}\sigma_x^2.
\end{aligned}$$

Now consider F and its derivative. Note

$$F(\bar{\omega}) = \int_0^{\bar{\omega}} dF(\omega) = \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} \exp^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx.$$

We make a change of variable similar to the one above:

$$\begin{aligned}
v &= \frac{x+\frac{1}{2}\sigma_x^2}{\sigma_x} \\
\bar{v} &= \frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x} \\
dv &= \frac{1}{\sigma_x}dx
\end{aligned}$$

so that

$$\begin{aligned}
F(\bar{\omega}) &= \int_0^{\bar{\omega}} dF(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}} \exp^{\frac{-v^2}{2}} dv \\
&= prob\left[v < \frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}\right]
\end{aligned}$$

Differentiate with respect to $\bar{\omega}$:

$$\begin{aligned}
F'(\bar{\omega}) &= \frac{1}{\bar{\omega}\sigma_x}\frac{1}{\sqrt{2\pi}} \exp^{\frac{-\left[\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}\right]^2}{2}} \\
&= \frac{1}{\bar{\omega}\sigma_x}\text{Standard Normal pdf}\left(\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}\right),
\end{aligned}$$

where the first equality uses Leibniz's rule.

3. No-monitoring Cost, no Sticky Price/Wage Version of the Model

When $\mu = 0$, the 5 equations that pertain to the entrepreneur part of the model reduce to:

$$E_t \frac{1 + R_{t+1}^k}{1 + R_t} = 0 \tag{3.1}$$

$$1 + R_t^k = \frac{r_t^k + (1 - \delta) q_t}{q_{t-1}} \pi_t \tag{3.2}$$

$$c_t + I_t + \Theta \frac{1 - \gamma_t}{\gamma_t} \left[n_{t+1} - w^e \right] = \epsilon_t \left(\frac{k_t}{\mu_z^*} \right)^\alpha h_t^{1-\alpha} - \phi \tag{3.3}$$

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_z} \left\{ R_t^k - R_{t-1} \right\} k_t q_{t-1} + w^e + \gamma_t \left(\frac{1 + R_{t-1}}{\pi_t \mu_z} \right) n_t. \tag{3.4}$$

$$\Gamma_t(\bar{\omega}_{t+1}) \left(1 + R_{t+1}^k \right) q_t k_{t+1} = (1 + R_t) B_{t+1} \tag{3.5}$$

The first equation is the first order condition associated with the optimal contract. We can derive it more simply in the case, $\mu = 0$. Again, we suppose that the entrepreneur cares only about the expected value of what he/she gets to keep at the end of the contract:

$$\max_{B_{t+1}, \{\bar{\omega}_{t+1}\}} E_t \overbrace{\left[1 - \Gamma_t(\bar{\omega}_{t+1}) \right]}^{\text{share of income from capital going to entrepreneur}} \left(1 + R_{t+1}^k \right) q_t k_{t+1},$$

subject to the state by state period $t+1$ zero profit condition:

$$\overbrace{\Gamma_t(\bar{\omega}_{t+1})}^{\text{share of entrepreneurial earnings received by bank}} \times \overbrace{\left(1 + R_{t+1}^k \right) q_t k_{t+1}}^{\text{entrepreneurial earnings}} = \overbrace{(1 + R_t) B_{t+1}}^{\text{obligations from bank to households in } t+1},$$

where

$$B_{t+1} = q_t k_{t+1} - n_{t+1}, \;\; \Gamma_t(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1} \left[1 - F_t(\bar{\omega}_{t+1}) \right] + G_t(\bar{\omega}_{t+1}).$$

This is the same as

$$\begin{aligned}
&\max_{B_{t+1}, \{\bar{\omega}_{t+1}\}} E_t \left\{ \overbrace{\left(1 + R_{t+1}^k \right) q_t k_{t+1}}^{\text{total earnings of capital}} - \overbrace{\Gamma_t(\bar{\omega}_{t+1}) \left(1 + R_{t+1}^k \right) q_t k_{t+1}}^{\text{part of earnings going to bank}} \right\} \\
= &\max_{B_{t+1}, \{\bar{\omega}_{t+1}\}} E_t \left\{ \left(1 + R_{t+1}^k \right) q_t k_{t+1} - (1 + R_t) \left(q_t k_{t+1} - n_{t+1} \right) \right\} \\
= &\max_{B_{t+1}, \{\bar{\omega}_{t+1}\}} E_t \left\{ \left(1 + R_{t+1}^k \right) \left(B_{t+1} + n_{t+1} \right) - (1 + R_t) B_{t+1} \right\} \qquad (3.6) \\
= &\max_{B_{t+1}, \{\bar{\omega}_{t+1}\}} E_t \left\{ \left[\left(1 + R_{t+1}^k \right) - (1 + R_t) \right] B_{t+1} + \left(1 + R_{t+1}^k \right) n_{t+1} \right\},
\end{aligned}$$

Note that this problem can be motivated in an even simpler way. Consider someone who has assets, n_{t+1}, and wishes to borrow B_{t+1} at a nominally state-non contingent interest rate, R_t. That person will choose B_{t+1} to maximize (3.6). As long as borrowing occurs (i.e., $B_{t+1} > 0$), it must be that

$$E_t R_{t+1}^k = R_t.$$

If we leave this like this, then (3.6) is sometimes positive, sometimes negative. How can we interpret this? The household is getting a non-state-contingent nominal payoff, $(1 + R_t) B_{t+1}$, and the projects collectively generate a state-contingent nominal payoff, $\left(1 + R^k_{t+1}\right)(B_{t+1} + n_{t+1})$. The 'incomplete markets' interpretation is that entrepreneurs in fact have a standard debt contract with banks, in which they pay a specified state-contingent interest rate in case they earn enough on their project and they turn over everything they have in case they do not earn enough. Under the 'complete markets' interpretation (see below) the entrepreneur also has a fixed nominal non-state-contingent interest rate with a bank, and the bank enters state-contingent financial markets with the household that allows it to pay the non-state-contingent amount, $(1 + R_t) B_{t+1}$. As a result, condition (3.6) only has to hold when summed across date $t + 1$ states of nature, where the different states are weighted by the household's marginal utility of currency in that state. Here, we pursue the incomplete markets interpretation. In part, this is because of evidence below that the equilibrium allocations are roughly the same in the two cases.

Under incomplete markets and standard debt contract interpretation, the zero profits condition of banks implies:

$$\Gamma_t(\bar{\omega}_{t+1}) \left(1 + R^k_{t+1}\right)(B_{t+1} + n_{t+1}) = (1 + R_t) B_{t+1},$$

or,

$$\Gamma_t(\bar{\omega}_{t+1}) = \frac{1 + R_t}{1 + R^k_{t+1}} \left(1 - \frac{n_{t+1}}{q_t k_{t+1}}\right), \tag{3.7}$$

where

$$\Gamma_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} \left\{1 - prob\left[v < \frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma^2_{x,t}}{\sigma_{x,t}}\right]\right\} + prob\left[x < \frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma^2_{x,t}}{\sigma_{x,t}} - \sigma_{x,t}\right]$$

Numerical experiments suggest that the function, $\Gamma_t(\bar{\omega}_{t+1})$, has a hump-shape in the neighborhood of its maximal value, when $\mu > 0$. However, in the $\mu = 0$ case considered here, $\Gamma_t(\bar{\omega}_{t+1})$ seems to be monotone increasing in $\bar{\omega}_{t+1}$. Thus, for given $\sigma^2_{x,t}$, there is a unique value of $\bar{\omega}_{t+1}$ that solves the above equation, if there is one at all. Since $\Gamma_t(\bar{\omega}_{t+1})$ is bounded above by unity, we cannot have R^k_{t+1} be too much smaller than R_t, for example. In steady state, it must be that $R^k_{t+1} = R_t$. In this case, a solution requires simply that the debt be positive.

When the state-contingent value of $\bar{\omega}_{t+1}$ which solves the zero profit condition has been found, then the state-contingent rate of interest, Z_{t+1}, on the contract can be determined from:

$$\bar{\omega}_{t+1} \frac{\left(1 + R^k_{t+1}\right) q_t k_{t+1}}{B_{t+1}} = Z_{t+1}. \tag{3.8}$$

The measure of entrepreneurs who pay this is $[1 - F_t(\bar{\omega}_{t+1})]$.

Also, to see the Fisher effect here, recall the alternative representation of the law of motion of net worth derived in (2.9):

$$n_{t+1} = \frac{\gamma_t}{\mu_z} \left\{ \left[r^k_t + (1 - \delta) q_t\right] k_t - \frac{1 + R_{t-1}}{\pi_t} B_t \right\} + w^e.$$

That is, next period's net worth is equal to the current period payoff on capital minus the real payments on the nominal debt, which are reduced by inflation when R_{t-1} is not contingent on the date t state. This corresponds to the Fisher effect on the debt. It is interesting to see how the intertemporal Euler equation is changed by this nominal rigidity. Making use of the fact, $(1+R_t) = E_t\left(1+R_{t+1}^k\right)$, we have that the household's intertemporal Euler equation is:

$$E_t\left(\frac{\beta}{\mu_z}\frac{\lambda_{z,t+1}}{\pi_{t+1}}\right)E_t\left(1+R_{t+1}^k\right) = \lambda_{z,t},$$

so,

$$E_t\frac{\beta}{\mu_z}\frac{\lambda_{z,t+1}}{\lambda_{z,t}\pi_{t+1}}\left(1+R_{t+1}^k\right) = 1 + cov_t\left(\frac{\beta}{\mu_z}\frac{\lambda_{z,t+1}}{\lambda_{z,t}\pi_{t+1}}, R_{t+1}^k\right). \tag{3.9}$$

3.1. No Idiosyncratic Uncertainty

Consider the case when there is no idiosyncratic uncertainty at the level of entrepreneurs. In this case,

$$G(\bar{\omega}), F(\bar{\omega}) = \begin{cases} 0 & \bar{\omega} < 1 \\ 1 & \bar{\omega} \geq 1 \end{cases},$$

so that $\Gamma(\bar{\omega})$ is a continuous function:

$$\Gamma(\bar{\omega}) = \bar{\omega}\left[1 - F(\bar{\omega})\right] + G(\bar{\omega}) = \begin{cases} \bar{\omega} & \bar{\omega} \leq 1 \\ 1 & \bar{\omega} \geq 1 \end{cases}.$$

For small enough fluctuations in aggregate shocks, $\Gamma(\bar{\omega}) < 1$, so that $\Gamma(\bar{\omega}) = \bar{\omega}$, and (3.7) implies:

$$\bar{\omega}_{t+1} = \frac{1+R_t}{1+R_{t+1}^k}\frac{B_{t+1}}{q_t k_{t+1}},$$

and, hence, using (3.8),

$$1 + R_t = Z_{t+1}.$$

for it is still true that the zero proft condition must be made to hold in each state. So, entrepreneurs, when there is no uncertainty, must simply pay interest equal to $1 + R_t$. For small enough shocks they can always afford this, since if $R_{t+1}^k < R_t$, they can simply dig into the earnings from their own net worth.

The equilibrium conditions in this case are:

$$\frac{1}{\lambda_f} = \frac{\lambda_w \frac{\psi_L h_t^{\sigma_L}}{\lambda_{z,t}}}{(1-\alpha)\,\epsilon_t}\left(\frac{\mu_z h_t}{k_t}\right)^{\alpha} \tag{3.10}$$

$$g_t + c_t + I_t + \Theta\frac{1-\gamma_t}{\gamma_t}\left[n_{t+1} - w^e\right] = \epsilon_t\left(\frac{k_t}{\mu_z}\right)^{\alpha} h_t^{1-\alpha} - \phi \tag{3.11}$$

$$k_{t+1} - (1-\delta)\mu_z^{-1}k_t = \left[1 - \frac{S''\mu_z^2}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t \tag{3.12}$$

$$E_t\{\beta\frac{1}{\mu_z}\lambda_{z,t+1}\frac{1+R_t}{\pi_{t+1}} - \lambda_{z,t}\} = 0 \tag{3.13}$$

$$E_t\left[\lambda_{z,t} - \frac{\mu_z}{c_t\mu_z - bc_{t-1}} + b\beta\frac{1}{\mu_z c_{t+1} - bc_t}\right] = 0 \tag{3.14}$$

$$\begin{aligned} &E_t\{\lambda_{zt}q_t\left[1 - \frac{S''\mu_z^2}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - S''\mu_z^2\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right] \\ &-\lambda_{zt} + \beta\lambda_{zt+1}q_{t+1}S''\mu_z^2\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\} = 0 \end{aligned} \tag{3.15}$$

$$r_t^k = \alpha\epsilon_t\left(\frac{\mu_z h_t}{k_t}\right)^{1-\alpha}\frac{1}{\lambda_f} \tag{3.16}$$

$$1 + R_t^k = \frac{r_t^k + (1-\delta)q_t}{q_{t-1}}\pi_t \tag{3.17}$$

$$E_t\left(1 + R_{t+1}^k\right) = 1 + R_t \tag{3.18}$$

$$n_{t+1} = \frac{\gamma_t}{\mu_z}\left\{\left[r_t^k + (1-\delta)q_t\right]k_t - \frac{1+R_{t-1}}{\pi_t}\left[q_{t-1}k_t - n_t\right]\right\} + w^e \tag{3.19}$$

This represents 10 equations in 11 unknowns: $h_t, k_{t+1}, c_t, I_t, \lambda_{z,t}, q_t, r_t^k, R_t^k, \pi_t, R_t, n_{t+1}$. Suppose we add an equation that characterizes monetary policy to close the system. Note that in this case, (3.19) plays no role in the dynamics of the model if $\Theta = 0$. Indeed, that equation and n_{t+1} can be dropped in the solution of the model, and it can be introduced later simply as a way to define net worth. So, the Fisher effect plays no role here. When $\Theta > 0$ then equation (3.19) cannot be dropped, because n_t also appears in the resource constraint.

To gain additional insight, consider the special case in which the household is the one doing the investment. In this case, we replace $(1+R_t)/\pi_{t+1}$ in the household's intertemporal Euler equation, (3.13), with $\left[r_{t+1}^k + (1-\delta)q_{t+1}\right]/q_t$. In this case, (3.10)-(3.16) represents 7 equations in 7 unknowns: $h_t, k_{t+1}, c_t, I_t, \lambda_{z,t}, q_t, r_t^k$. I suspect that, up to linearization, there is no distinction between these two systems when $\Theta = 0$. One problem is that entrepreneurial net worth does not exert a drag on the ability of entrepreneurs to buy capital. Without this, it's not clear why an entrepreneur who, say, has his net worth reduced because of a jump in the price level, would be affected in how much he borrows. How much or how little investment he does should be dictated by investment opportunities. Inspection of the Euler equation, (3.9), indicates one slight effect of having the entrepreneurs do investment, which is that they are not risk averse. But, given that risk aversion plays very little role in household investment, giving the task of investment over to risk averse entrepreneurs should not change things very much (and, perhaps not at all in the first order approximation).

3.2. With Idiosyncratic Uncertainty

When there is idiosyncratic uncertainty and $\mu = 0$, then the equations of the system are still the ones in the previous subsection. In particular, the Fisher effect and net worth play no role in the dynamics of the system if $\Theta = 0$. In this case, the law of motion of net worth simply defines net worth and plays no role in model dynamics. When $\Theta > 0$, then net worth enters as though it were a lump sum tax on ordinary households and creates a wealth effect on them.

3.3. No Frictions, $\mu = 0$

We take this down to the simplest version of our model, no sticky prices/wages, no adjustment costs, no habit. Just the financial frictions.

$$\frac{1}{\lambda_f} = \frac{\lambda_w \psi_L h_t^{\sigma_L} c_t}{(1-\alpha)\,\epsilon_t} \left(\frac{\mu_z h_t}{k_t} \right)^{\alpha} \tag{3.20}$$

$$g_t + c_t + I_t + \Theta \frac{1-\gamma_t}{\gamma_t} \left[n_{t+1} - w^e \right] = \epsilon_t \left(\frac{k_t}{\mu_z} \right)^{\alpha} h_t^{1-\alpha} - \phi \tag{3.21}$$

$$k_{t+1} - (1-\delta)\mu_z^{-1} k_t = I_t \tag{3.22}$$

$$E_t \frac{\beta}{\mu_z c_{t+1}} \frac{1+R_t}{\pi_{t+1}} = \frac{1}{c_t} \tag{3.23}$$

$$r_t^k = \alpha \epsilon_t \left(\frac{\mu_z h_t}{k_t} \right)^{1-\alpha} \frac{1}{\lambda_f} \tag{3.24}$$

$$E_t \left[r_{t+1}^k + (1-\delta) \right] \pi_{t+1} = 1 + R_t \tag{3.25}$$

$$n_{t+1} = \frac{\gamma_t}{\mu_z} \left\{ \left[r_t^k + 1 - \delta \right] k_t - \frac{1+R_{t-1}}{\pi_t} \left[k_t - n_t \right] \right\} + w^e \tag{3.26}$$

This is 7 equations in 8 unknowns: $h_t, k_{t+1}, c_t, I_t, R_t, \pi_t, r_t^k, n_{t+1}$. Note that when $\Theta = 0$ (3.20)-(3.25) represents 6 equations in the 7 unknowns, $h_t, k_{t+1}, c_t, I_t, R_t, \pi_t, r_t^k$. These could be solved by adding a Taylor rule monetary policy. In this case, (3.26) would simply define n_t, but would have no impact on the dynamics of the variables.

3.4. Steady State

We now have a total of 11 equations (including the 5 in (3.1)-(3.5)) in the 7 previous unknowns plus 5 additional $\Gamma_t(\bar{\omega}_{t+1})$, R_t^k, π_t, d_t, n_t variables. Now, the system requires an additional equation to solve. That equation is provided by the monetary policy rule.

We discuss the computation of the steady state of the model. We have:

$$R^k = R.$$

The households' intertemporal condition on nominal debt implies:

$$1 + R = \frac{\mu_z \pi}{\beta}.$$

The intertemporal equation on investment implies $q = 1$. The sticky wage equilibrium conditions are:

$$\begin{aligned} F_w &= \frac{\lambda_z \frac{h}{\lambda_w}}{1-\beta\xi_w} \\ K_w &= \frac{1}{\psi_L}\tilde{w}F_w \\ K_w &= \frac{h^{1+\sigma_L}}{1-\beta\xi_w}, \end{aligned}$$

so that

$$\frac{h^{1+\sigma_L}}{1-\beta\xi_w} = \frac{1}{\psi_L}\tilde{w}\frac{\lambda_z \frac{h}{\lambda_w}}{1-\beta\xi_w} \rightarrow \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z} = w$$

The sticky price equilibrium conditions imply $s = 1/\lambda_f$. Efficient employment decisions by firms then imply:

$$\frac{1}{\lambda_f} = \frac{w}{(1-\alpha)}\left(\frac{\mu_z h_t}{k_t}\right)^{\alpha},$$

which can be put together with the household condition to obtain:

$$\frac{1}{\lambda_f} = \frac{\lambda_w \frac{\psi_L h_t^{\sigma_L}}{\lambda_{z,t}}}{(1-\alpha)}\left(\frac{\mu_z h_t}{k_t}\right)^{\alpha}.$$

We have that

$$\frac{1+R^k}{\pi} = r^k + 1 - \delta = \frac{\mu_z}{\beta},$$

where efficiency in the use of capital by intermediate good firms implies:

$$r^k = \alpha\left(\frac{\mu_z h}{k}\right)^{1-\alpha}\frac{1}{\lambda_f}.$$

Finally, the resource constraint, law of motion of net worth and zero profit condition on banks correspond to:

$$\begin{aligned} c + I + \Theta\frac{1-\gamma}{\gamma}\left[n - w^e\right] &= \left(\frac{k}{\mu_z}\right)^{\alpha} h^{1-\alpha} - \phi \\ n &= \frac{\gamma}{\mu_z}\left[r^k + 1 - \delta\right]k + w^e - \gamma\left(\frac{1+R}{\pi\mu_z}\right)B \\ \overbrace{(1+R)\left(\bar{\omega}\left[1-F(\bar{\omega})\right] + G\left(\bar{\omega}\right)\right)k}^{\text{total earnings of entrepreneurs}} &= \overbrace{(1+R)\left(k-n\right)}^{\text{total payments to households}}. \end{aligned}$$

We now describe a numerical algorithm for finding the steady state. First,

$$\alpha\left(\frac{\mu_z h}{k}\right)^{1-\alpha}\frac{1}{\lambda_f} = \frac{\mu_z}{\beta} - (1-\delta)$$

delivers the labor/capital ratio:

$$\frac{h}{k} = \frac{1}{\mu_z}\left[\lambda_f \frac{\frac{\mu_z}{\beta} - (1-\delta)}{\alpha}\right]^{\frac{1}{1-\alpha}}.$$

Manipulating the law of motion of net worth, we obtain:

$$\frac{n}{k} = \frac{w^e}{k} + \frac{\gamma}{\beta}\frac{n}{k} = \frac{\frac{w^e}{k}}{1-\frac{\gamma}{\beta}}.$$

Also,

$$\frac{n}{k} - \frac{w^e}{k} = \frac{w^e}{k}\left[\frac{1}{1-\frac{\gamma}{\beta}} - 1\right] = \frac{w^e}{k}\frac{\frac{\gamma}{\beta}}{1-\frac{\gamma}{\beta}}$$

We work out the implications of the steady state zero profit condition for ϕ. Thus,

$$\begin{aligned}
\left(\frac{k}{\mu_z}\right)^{\alpha} h^{1-\alpha} &= f_k k + f_h h \\
f_k &= \alpha \frac{1}{\mu_z}\left(\frac{\mu_z h}{k}\right)^{1-\alpha},\ f_h = (1-\alpha)\left(\frac{k}{\mu_z h}\right)^{\alpha} \\
r^k &= \alpha\left(\frac{\mu_z h}{k}\right)^{1-\alpha}\frac{1}{\lambda_f} = \frac{\mu_z f_k}{\lambda_f},\ f_k = \frac{\lambda_f}{\mu_z} r^k \\
\tilde{w} &= \frac{1}{\lambda_f}(1-\alpha)\left(\frac{\mu_z h}{k}\right)^{-\alpha} = \frac{1}{\lambda_f} f_h, \\
&= \lambda_f\left[\frac{1}{\mu_z} r^k k + \tilde{w} h\right] - \phi
\end{aligned}$$

Then,

$$\begin{aligned}
y \overbrace{=}^{\text{zero ss profits}} \frac{1}{\mu_z} r^k k + \tilde{w} h &= \frac{1}{\lambda_f}\left[f_k k + f_h h\right] \\
&= \frac{1}{\lambda_f}\left[y + \phi\right] \\
(\lambda_f - 1)\, y &= \phi,
\end{aligned}$$

We conclude:

$$\begin{aligned}
y &= \left(\frac{k}{\mu_z}\right)^{\alpha} h^{1-\alpha} - \phi \\
&= \left(\frac{k}{\mu_z}\right)^{\alpha} h^{1-\alpha} - (\lambda_f - 1)\, y,
\end{aligned}$$

or,

$$y = \frac{1}{\lambda_f}\left(\frac{k}{\mu_z}\right)^{\alpha} h^{1-\alpha}$$

Substituting the latter into the resource constraint:

$$g+c+I+\Theta\frac{1-\gamma}{\gamma}\left[n-w^{e}\right]=\frac{1}{\lambda_{f}}\left(\frac{k}{\mu_{z}}\right)\left(\frac{\mu_{z}h}{k}\right)^{1-\alpha}$$

or, since $\eta_{g}y=g$,

$$c+I+\Theta\frac{1-\gamma}{\gamma}\left[n-w^{e}\right]=\frac{1-\eta_{g}}{\lambda_{f}}\left(\frac{k}{\mu_{z}}\right)\left(\frac{\mu_{z}h}{k}\right)^{1-\alpha}$$

Substituting the steady state capital accumulation equation:

$$\left[1-(1-\delta)\mu_{z}^{-1}\right]k=I,$$

into the resource constraint:

$$\frac{c}{k}+\left[1-(1-\delta)\mu_{z}^{-1}\right]+\Theta\frac{1-\gamma}{\gamma}\left[\frac{n}{k}-\frac{w^{e}}{k}\right]=\frac{1-\eta_{g}}{\lambda_{f}}\frac{1}{\mu_{z}}\left(\frac{\mu_{z}h}{k}\right)^{1-\alpha}.$$

Substituting out the simplified version of the law of motion for net worth:

$$\frac{c}{k}+\Theta\frac{1-\gamma}{\gamma}\frac{\frac{\gamma}{\beta}}{1-\frac{\gamma}{\beta}}\frac{w^{e}}{k}=\frac{1-\eta_{g}}{\lambda_{f}}\frac{1}{\mu_{z}}\left(\frac{\mu_{z}h}{k}\right)^{1-\alpha}-\left[1-(1-\delta)\mu_{z}^{-1}\right]$$

The relationship between the marginal rate of substitution and the marginal product of labor is, after rearranging,

$$\frac{1}{\lambda_{f}\lambda_{w}}=\frac{\frac{\psi_{L}h^{\sigma_{L}}}{\lambda_{z}}}{(1-\alpha)}\left(\frac{\mu_{z}h}{k}\right)^{\alpha}$$

and we also have:

$$c\frac{\mu_{z}-b}{\mu_{z}-b\beta}=\frac{1}{\lambda_{z}}.$$

Combining the latter two:

$$\frac{1-\alpha}{\lambda_{f}\lambda_{w}\psi_{L}\left(\frac{\mu_{z}h}{k}\right)^{\alpha}}\frac{\mu_{z}-b\beta}{\mu_{z}-b}=h^{\sigma_{L}}c=\left(\frac{\mu_{z}h}{k}\right)^{\sigma_{L}}\left(\frac{k}{\mu_{z}}\right)^{\sigma_{L}}\frac{k}{\mu_{z}}\frac{c}{k}\mu_{z},$$

or,

$$\frac{1-\alpha}{\mu_{z}\lambda_{f}\lambda_{w}\psi_{L}\left(\frac{\mu_{z}h}{k}\right)^{\alpha+\sigma_{L}}}\frac{\mu_{z}-b\beta}{\mu_{z}-b}\left(\frac{k}{\mu_{z}}\right)^{-(1+\sigma_{L})}=\frac{c}{k}.$$

substitute this into the resource constraint:

$$\frac{\frac{1-\alpha}{\mu_{z}\lambda_{f}\lambda_{w}\psi_{L}\left(\frac{\mu_{z}h}{k}\right)^{\alpha+\sigma_{L}}}\frac{\mu_{z}-b\beta}{\mu_{z}-b}}{\left(\frac{k}{\mu_{z}}\right)^{(1+\sigma_{L})}}+\Theta\frac{1-\gamma}{\gamma}\frac{\frac{\gamma}{\beta}}{1-\frac{\gamma}{\beta}}\frac{w^{e}}{k}=\frac{1-\eta_{g}}{\lambda_{f}}\frac{1}{\mu_{z}}\left(\frac{\mu_{z}h}{k}\right)^{1-\alpha}-\left[1-(1-\delta)\mu_{z}^{-1}\right]$$

which is a single equation in the one unknown, k. Alternatively,

$$\frac{a}{\left(\frac{k}{\mu_z}\right)^{(1+\sigma_L)}}+\Theta\frac{1-\gamma}{\gamma}\frac{\frac{\gamma}{\beta}}{1-\frac{\gamma}{\beta}}\frac{w^e}{k}=b,$$

where

$$\begin{aligned} a &= \frac{1-\alpha}{\mu_z\lambda_f\lambda_w\psi_L\left(\frac{\mu_z h}{k}\right)^{\alpha+\sigma_L}}\frac{\mu_z-b\beta}{\mu_z-b} \\ b &= \frac{1-\eta_g}{\lambda_f}\frac{1}{\mu_z}\left(\frac{\mu_z h}{k}\right)^{1-\alpha}-\left[1-(1-\delta)\mu_z^{-1}\right] \end{aligned}$$

Obviously, there exists a solution for k, and that solution is unique. If $\Theta=0$, then it can be solved for analytically. Since we work with a small value of Θ, it is useful to consider, as a starting point for nonlinear computation, the steady state value of k when $\Theta=0$:

$$k=\mu_z\left(\frac{a}{b}\right)^{\frac{1}{1+\sigma_L}}.$$

The actual value of k will be somewhat larger than this.

Given k all the other variables substituted out in the previous derivations can be computed. We obtain $\bar{\omega}$ from the bank zero-profit condition:

$$\bar{\omega}\left[1-F(\bar{\omega})\right]+G\left(\bar{\omega}\right)=\frac{k-n}{k}. \tag{3.27}$$

This is the steady state debt to asset ratio of entrepreneurs. This determines the steady state share of entrepreneurial profits going to banks. Recall,

$$\begin{aligned} F(\bar{\omega}) &= prob\left[v<\frac{\log\left(\bar{\omega}\right)+\frac{1}{2}\sigma_x^2}{\sigma_x}\right] \\ G\left(\bar{\omega}\right) &= prob\left[x<\frac{\log\left(\bar{\omega}\right)+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x\right]. \end{aligned}$$

We find $\bar{\omega}$ as follows. First, set a calibrated value of $F\left(\bar{\omega}\right)$, the bankruptcy rate. Given $\bar{\omega}$, this allows us to compute σ_x^2. Given $\bar{\omega}$ and σ_x^2 we can compute $G\left(\bar{\omega}\right)$ and, hence $\bar{\omega}\left[1-F(\bar{\omega})\right]+G\left(\bar{\omega}\right)$. Then, adjust $\bar{\omega}$ until (3.27) is satisfied.

4. The Fisherian Debt-Deflation Hypothesis

We wish to diagnose the role of the assumption that payments to households are non-state contingent in nominal terms. We do this by exploring the BGG version of the model in which the payment on households' bank deposits is non-state contingent in real terms. Thus, suppose that instead of earning gross nominal return, $1+R_t$, from t to $t+1$ households instead earn gross nominal return,

$$F_t\pi_{t+1},$$

from t to $t+1$. Here, F_t denotes the real return from t to $t+1$, which is non-state contingent in real terms. With two exceptions, we substitute $1+R_t$ with $F_t\pi_{t+1}$ everywhere. The two exceptions are the Taylor rule, where we continue to assume a non-state contingent nominal rate of interest is 'controlled'. To ensure that that rate of interest is well defined, we keep equation (10). We add an equation for household deposits:

$$(10)' E_t\{\beta \frac{1}{\mu_{z,t+1}} \lambda_{z,t+1} F_t - \lambda_{z,t}\} = 0.$$

We must change the relevant equations associated with the entrepreneur. The zero profit condition becomes:

$$(16)' \ \Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t) = \frac{F_{t-1}\pi_t}{1+R_t^k}\left(1 - \frac{n_t}{q_{t-1}k_t}\right).$$

The optimality condition becomes:

$$(17)' E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R_{t+1}^k}{F_t\pi_{t+1}} + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})}\left[\frac{1+R_{t+1}^k}{F_t\pi_{t+1}}\left(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right) - 1\right]\right\} = 0$$

and the law of motion of net worth becomes:

$$(18)' n_{t+1} = \frac{\gamma_t}{\pi_t \mu_z^*}\left\{1 + R_t^k - F_{t-1}\pi_t - \mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\right\} k_t q_{t-1} + w^e + \gamma_t \frac{F_{t-1}}{\mu_z} n_t$$

5. Complete Markets Version of the Model

We replace the state-by-state zero profit condition, (2.2), by

$$\begin{aligned} & E_t m_{t+1}\left[\left[1 - F_t\left(\bar{\omega}_{t+1}\right)\right] Z_{t+1} B_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)\left(1+R_{t+1}^k\right) q_t k_{t+1}\right] \\ = \ & E_t m_{t+1}\left(1+R_t\right) B_{t+1} = B_{t+1}, \end{aligned}$$

using (1.21). Here, m_{t+1} is the relative price of consumption in period t and each state of nature in period $t+1$, which is treated as exogenous by the bank. In equilibrium, this relative price is related to the household's intertemporal marginal rate of substitution in consumption as follows:

$$m_{t+1} = \frac{\beta\lambda_{t+1}}{\lambda_t} = \frac{\beta\lambda_{z,t+1}}{\lambda_{z,t}\pi_{t+1}\mu_{z,t+1}}.$$

In effect, this specification of the zero profit condition allows the bank to have non-zero profits in each individual state of nature, as long as the zero profit condition is satisfied. Positive and negative cash positions in different states of nature correspond to transfers between banks and households. Thus, we are implicitly allowing households to participate in state contingent markets for money, in addition to holding deposits with banks.

We suppose that Z_{t+1}, the nominal interest rate paid by non-bankrupt entrepreneurs, is not contingent upon the date $t+1$ state of nature. Thus, the standard debt contract offered in period t is associated with a single Z_{t+1} and a single B_{t+1}. There is a sequence of $\bar{\omega}_{t+1}$'s across period $t+1$ states of nature which is determined by (2.1). To derive the equilibrium condition associated with the optimal debt contract, it is convenient to substitute out for $\bar{\omega}_{t+1}$ using (2.1):

$$E_t m_{t+1}\left\{\left[1-F_t\left(\frac{Z_{t+1}B_{t+1}}{\left(1+R_{t+1}^k\right)q_t k_{t+1}}\right)\right]Z_{t+1}\frac{B_{t+1}}{q_t k_{t+1}}+(1-\mu)\int_0^{\frac{Z_{t+1}B_{t+1}}{\left(1+R_{t+1}^k\right)q_t k_{t+1}}}\omega dF_t(\omega)\left(1+R_{t+1}^k\right)\right\}$$
$$=\frac{B_{t+1}}{q_t k_{t+1}}$$

Making use of the definition of the asset to net worth ratio, ϱ_t, in (2.5):

$$E_t m_{t+1}\left[\left[1-F_t\left(\frac{A_t}{1+R_{t+1}^k}\right)\right]A_t+(1-\mu)\int_0^{\frac{A_t}{1+R_{t+1}^k}}\omega dF_t(\omega)\left(1+R_{t+1}^k\right)\right]=1-\frac{1}{\varrho_t},$$

where

$$A_t\equiv Z_{t+1}\left(1-\frac{1}{\varrho_t}\right)=Z_{t+1}\frac{B_{t+1}}{q_t k_{t+1}}(=\bar{\omega}_{t+1}\left(1+R_{t+1}^k\right)),$$

and, recall,

$$\varrho_t\equiv\frac{q_t k_{t+1}}{n_{t+1}}.$$

The standard debt contract can now be expressed in terms of two parameters, ϱ_t and A_t. Competition among banks ensures that these two parameters are selected to maximize entrepreneurial utility subject to the zero profit condition. The Lagrangian representation of the problem is:

$$\max_{\varrho_t,A_t}E_t\{\frac{\left[1-\Gamma_t\left(\frac{A_t}{1+R_{t+1}^k}\right)\right]\left(1+R_{t+1}^k\right)}{1+R_t}\varrho_t$$
$$+\eta_t\left\{E_t m_{t+1}\left[\left[1-F_t\left(\frac{A_t}{1+R_{t+1}^k}\right)\right]A_t+(1-\mu)\int_0^{\frac{A_t}{1+R_{t+1}^k}}\omega dF_t(\omega)\left(1+R_{t+1}^k\right)\right]+\frac{1}{\varrho_t}-1\right\}\}.$$

Since the zero profit condition associated with the period t problem is now represented by a single equation, there is only one multiplier, η_t, for each date. The first order necessary condition associated with the optimal ϱ_t is

$$\eta_t=(\varrho_t)^2E_t\left(\frac{\left[1-\Gamma_t\left(\frac{A_t}{1+R_{t+1}^k}\right)\right]\left(1+R_{t+1}^k\right)}{1+R_t}\right).$$

The first order necessary condition associated with the optimal A_t is:[3]

$$E_t \frac{-\Gamma_t'\left(\frac{A_t}{1+R_{t+1}^k}\right)}{1+R_t}\varrho_t + \eta_t E_t m_{t+1}[-F_t'\left(\frac{A_t}{1+R_{t+1}^k}\right)\frac{A_t}{1+R_{t+1}^k} + 1 - F_t\left(\frac{A_t}{1+R_{t+1}^k}\right) + (1-\mu)\frac{A_t}{1+R_{t+1}^k}F_t'\left(\frac{A_t}{1+R_{t+1}^k}\right)] = 0,$$

or,

$$E_t\left\{\eta_t \frac{\beta\lambda_{z,t+1}}{\lambda_{z,t}\pi_{t+1}\mu_{z,t+1}}\left[1-F_t\left(\frac{A_t}{1+R_{t+1}^k}\right) - \mu F_t'\left(\frac{A_t}{1+R_{t+1}^k}\right)\frac{A_t}{1+R_{t+1}^k}\right] - \frac{\Gamma_t'\left(\frac{A_t}{1+R_{t+1}^k}\right)}{1+R_t}\varrho_t\right\} = 0$$

We summarize the equilibrium conditions associated with the standard debt contract in terms of the notation of the other sections:

$$0 = E_t\{\eta_t \frac{\beta\lambda_{z,t+1}}{\lambda_{z,t}\pi_{t+1}\mu_{z,t+1}}\left[1-F_t\left(\frac{A_t}{1+R_{t+1}^k}\right) - \mu F_t'\left(\frac{A_t}{1+R_{t+1}^k}\right)\frac{A_t}{1+R_{t+1}^k}\right] - \frac{1-F_t\left(\frac{A_t}{1+R_{t+1}^k}\right)}{1+R_t}\varrho_t\} \quad (5.1)$$

$$\eta_t = (\varrho_t)^2 E_t\left(\frac{\left[1-\Gamma_t\left(\frac{A_t}{1+R_{t+1}^k}\right)\right]\left(1+R_{t+1}^k\right)}{1+R_t}\right)$$

$$1-\frac{1}{\varrho_t} = E_t \frac{\beta\lambda_{z,t+1}}{\lambda_{z,t}\pi_{t+1}\mu_{z,t+1}}\left(1+R_{t+1}^k\right)\left[\Gamma_t\left(\frac{A_t}{1+R_{t+1}^k}\right) - \mu G_t\left(\frac{A_t}{1+R_{t+1}^k}\right)\right], \quad (5.2)$$

where

$$A_t = \bar{\omega}_{t+1}\left(1+R_{t+1}^k\right).$$

The three equilibrium conditions associated with the financial frictions are the law of motion of net worth, (2.8), the optimality condition (5.1) and the zero profit condition, (5.2). In addition, the resource constraint is (2.10).

6. Simplified Equilibrium Conditions

Here is a summary of the equations. They are expressed in simplified form. There are no adjustment costs in capital, no sticky wages, and no habit persistence in consumption. The three versions of the model are differentiated.

[3] We have made use of the result,

$$G'(x) = g(f(x))f'(x),$$

when

$$G(x) = \int^{f(x)} g(\omega)\,d\omega.$$

6.1. CEE Model

The pricing setting equations are as before (we set $p_t^* = 1$ because this is without loss of generality when linearizing and $w_t^* = 1$ because there are no wage setting frictions):

$$(2) E_t \left\{ \lambda_{z,t} \epsilon_t \left(\frac{k_t}{\mu_z} \right)^{\alpha} h_t^{1-\alpha} + \left(\frac{\pi_t^{\iota 2} \bar{\pi}^{1-\iota 2}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0,$$

and

$$(3) \lambda_{z,t} \lambda_f \left[\epsilon_t \left(\frac{k_t}{\mu_z} \right)^{\alpha} h_t^{1-\alpha} - \phi \right] s_t +$$

$$\beta \xi_p \left(\frac{\pi_t^{\iota 2} \bar{\pi}^{1-\iota 2}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \left[\frac{1 - \xi_p \left(\frac{\pi_t^{\iota 2} \bar{\pi}^{1-\iota 2}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} F_{p,t+1} - F_{p,t} \left[\frac{1 - \xi_p \left(\frac{\pi_{t-1}^{\iota 2} \bar{\pi}^{1-\iota 2}}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1 - \xi_p} \right]^{1-\lambda_f} = 0.$$

When linearized about steady state, these reduce to the usual Calvo equation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \gamma \hat{s}_t + \hat{\lambda}_{f,t}, \ \gamma \equiv \frac{\left(1 - \xi_p\right)\left(1 - \beta \xi_p\right)}{\xi_p}.$$

Setting $\xi_w = 0$, (4) and (5) reduce to the usual condition that the wage (scaled by $P_t z_t$) is a markup over marginal cost:

$$\tilde{w}_t = \lambda_w \frac{\psi_L h_t^{\sigma_L}}{\lambda_{z,t}}, \ \lambda_{z,t} \equiv \frac{1}{c_t},$$

where c_t is consumption, scaled by z_t. Real marginal cost:

$$s_t = \frac{\tilde{w}_t}{(1-\alpha) \epsilon_t} \left(\frac{\mu_z h_t}{k_t} \right)^{\alpha},$$

where k_t is the stock of capital, scaled by z_{t-1}. Resource constraint:

$$c_t + I_t = \epsilon_t \left(\frac{k_t}{\mu_z} \right)^{\alpha} h_t^{1-\alpha}$$

where I_t is investment, scaled by z_t and

$$k_{t+1} - (1-\delta) \mu_z^{-1} k_t = I_t.$$

Equation defining the nominal non-state contingent rate of interest:

$$E_t \{ \beta \frac{1}{\pi_{t+1} \mu_z} \lambda_{z,t+1} (1 + R_t) - \lambda_{z,t} \} = 0$$

The capital first order condition:

$$E_t \left\{ -\lambda_{z,t} + \lambda_{z,t+1} \frac{\beta}{\mu_z} \left(1 + R_{t+1}^k\right) \right\} = 0,$$

where R^k_{t+1} denotes the rate of return on capital:

$$1+R^k_t = r^k_t + 1 - \delta,\ r^k_t = \alpha\epsilon_t\left(\frac{\mu_z h_t}{k_t}\right)^{1-\alpha} s_t.$$

The monetary policy rule:

$$\begin{aligned}\log(1+R_t) &= (1-\rho)\log(1+R) + \rho\log(1+R_{t-1}) \\ &+ \frac{1}{1+R}(1-\rho)\tilde{a}_p\pi\log\frac{\pi_{t+1}}{\pi} + (1-\rho)\tilde{a}_y\frac{1}{4(1+R)}\log\frac{y_t}{y} + \frac{1}{400(1+R)}x^p_t,\end{aligned}$$

where y_t denotes GDP (i.e., $c_t + I_t$), scaled by z_t.

6.2. CEE+BGG

We drop the household's capital accumulation Euler equation. We add the following equations. Bank zero profit condition:

$$[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]\frac{q_t k_{t+1}}{n_{t+1}}\left(1+R^k_{t+1}\right) = (1+R_t)\left(\frac{q_t k_{t+1}}{n_{t+1}} - 1\right)$$

which must hold in each realized $t+1$ state of nature. Optimality condition for entrepreneurial loan contract:

$$E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R_t} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}\left[\frac{1+R^k_{t+1}}{1+R_t}\left(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right) - 1\right]\right\} = 0.$$

Law of motion of entrepreneurial net worth, n_{t+1} :

$$n_{t+1} = \frac{\gamma_t}{\pi_t\mu_z}\left\{R^k_t - R_{t-1} - \mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R^k_t\right)\right\}k_t q_{t-1} + w^e + \gamma_t\left(\frac{1+R_{t-1}}{\pi_t\mu_z}\right)n_t.$$

The resource constraint replaced by:

$$d_t + c_t + I_t + \Theta\frac{1-\gamma_t}{\gamma_t}[n_{t+1} - w^e] = \epsilon_t\left(\frac{k_t}{\mu^*_z}\right)^\alpha L^{1-\alpha}_t,$$

where

$$d_t = \frac{\mu G(\bar{\omega}_t)\left(1+R^k_t\right)q_{t-1}k_t}{\mu^*_z}\frac{1}{\pi_t}.$$

6.3. CEE+BGG-Fisher

The households are given an additional equation:

$$E_t\{\beta\frac{1}{\mu_z}\lambda_{z,t+1}F_t - \lambda_{z,t}\} = 0.$$

The zero profit condition on banks becomes:

$$\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t) = \frac{F_{t-1}\pi_t}{1+R^k_t}\left(1 - \frac{n_t}{q_{t-1}k_t}\right).$$

The optimality condition becomes:

$$E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R_{t+1}^k}{F_t\pi_{t+1}}+\frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1})-\mu G_t'(\bar{\omega}_{t+1})}\left[\frac{1+R_{t+1}^k}{F_t\pi_{t+1}}\left(\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})\right)-1\right]\right\}=0$$

and the law of motion of net worth becomes:

$$n_{t+1}=\frac{\gamma_t}{\pi_t\mu_z^*}\left\{1+R_t^k-F_{t-1}\pi_t-\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^k\right)\right\}k_t q_{t-1}+w^e+\gamma_t\frac{F_{t-1}}{\mu_z}n_t$$

7. Results

We first describe the results relative to a benchmark set of frictions. We then examine the role of those frictions by shutting them down.

7.1. Benchmark Frictions

We used the following parameter values:

$$\begin{aligned}
\xi_p &= 0.75,\ \lambda_w=1.05,\ \sigma_L=1,\ b=0.56,\ \beta=1.004^{-.25},\ \psi_L=110,\\
F(\bar{\omega}) &= 0.026,\ \mu=0.01,\ \gamma=1-0.022,\ \Theta=0.02,\\
\frac{w^e}{c+I+g} &= 0.014,\ \Theta\frac{1-\gamma}{\gamma}\frac{(n-w^e)}{y}=0.0006,\ \mu_z=1.0053,\\
\lambda_f &= 1.20,\ \alpha=0.36,\ \delta=0.02,\\
\iota &= 0\text{ (i.e., no indexation to lag inflation)},\ \tilde{a}_p=1.85,\ \tilde{a}_y=0.20,\\
\tilde{\rho} &= 0.8,\ 100\left(\pi^4-1\right)=3.5,\ \rho=0.9729,\\
\iota_w &= 0.40\text{ (wage indexation to lag inflation)},\ S''=37.43,\\
\frac{c}{c+g+I} &= 0.47,\ \frac{I}{c+g+I}=0.33,\ \frac{\Phi}{c+I+g}=0.20,\ \frac{g}{c+g+I}=0.20
\end{aligned}$$

Consider the dynamic response to a technology shock in the various versions of the model considered. The shock drives up the state of technology by 1 percent, which then decays back to steady state according to a geometric pattern with coefficient 0.9729.

Percent Response in GDP to 1 percent Technology Shock

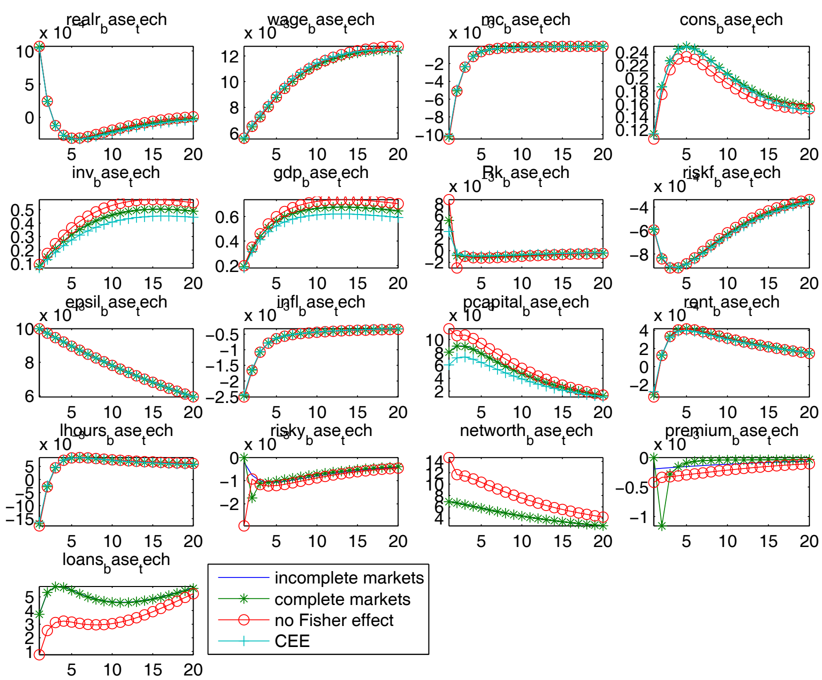
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	0.18	0.52	0.61	0.62	0.61	0.58	0.55
Incomplete	0.19	0.56	0.66	0.68	0.67	0.64	0.60
Incomplete/No Fisher	0.20	0.60	0.71	0.74	0.73	0.70	0.65
Complete	0.19	0.56	0.66	0.68	0.67	0.64	0.60

There are several things worth noting about these results. Whether markets are complete or not makes virtually no difference. Also, the model with financial frictions (with or without incomplete markets) produces relatively similar response in output compared with the CEE model. When the Fisher effect is dropped from the incomplete markets version of the model, then the effects on output are stronger. This is consistent with the view that the fall in the price level associated with a positive technology shock reallocates wealth away from entrepreneurs. This inhibits entrepreneurs' ability to buy capital, and acts on a drag on output by slowing investment.

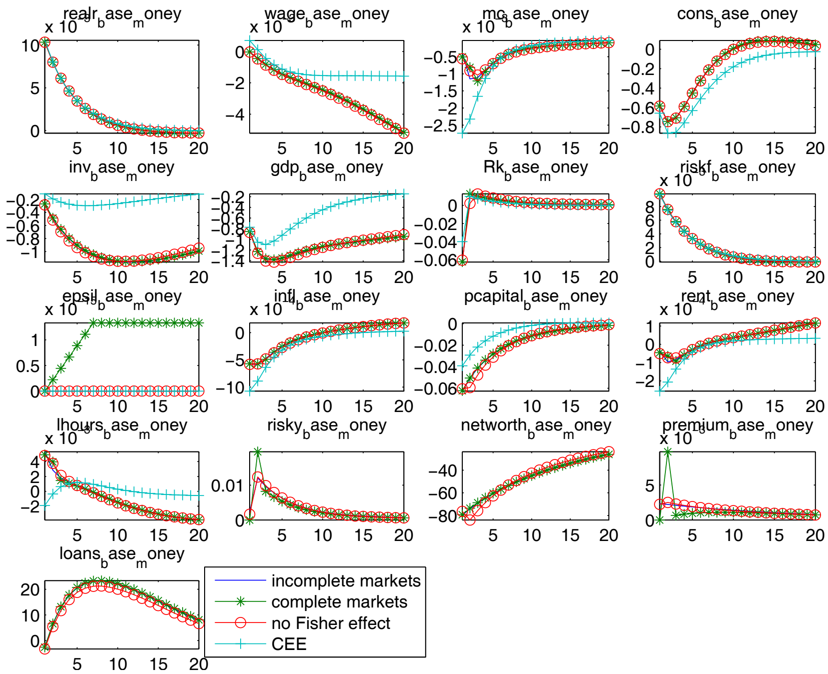
Consider now the effects of a monetary policy shock. We perturb x_t^p in (1.25) by .01. This represents a 4 percentage point (400 basis points!) shock to the interest rate.

	Percent Response in GDP to Monetary Policy Shock						
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	-0.77	-0.92	-0.52	-0.30	-0.19	-0.13	-0.10
Incomplete	-0.86	-1.36	-1.18	-1.07	-1.00	-0.93	-0.85
Incomplete/No Fisher	-0.87	-1.39	-1.19	-1.06	-0.97	-0.89	-0.80
Complete	-0.85	-1.35	-1.17	-1.06	-1.00	-0.92	-0.84

Note that now the financial frictions are more important. In the 17th quarter, the effects on output are 5 times larger than they are in CEE. Once again, we see that whether markets are modeled as complete or not makes no difference. Following is the graph of the dynamic response of gdp and other variables, to a technology shock. With some exceptions, all objects are in deviation from steady state. The exceptions are consumption, investment, output, net worth and loans. Their deviation from steady state is multiplied by 100 and divided by steady state gdp.



Following is the response of the variables to a monetary policy shock:



Interestingly, the effects of the financial frictions that we describe here are smaller for larger monitoring costs, μ. When we set $\mu = 0.15$ ($\mu = 0.12$ is the calibrated value used in BGG) we obtain the following response to a monetary policy shock:

Percent Response in GDP to Monetary Policy Shock, $\mu = 0.15$

	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	-0.77	-0.92	-0.52	-0.30	-0.19	-0.13	-0.10
Incomplete	-0.93	-1.11	-0.65	-0.41	-0.30	-0.24	-0.19
Incomplete/No Fisher	-0.93	-1.11	-0.63	-0.39	-0.27	-0.20	-0.16
Complete	-0.92	-1.09	-0.63	-0.41	-0.30	-0.24	-0.20

There is now a much smaller effect from the financial frictions.

It is even possible to reverse the impact of the financial frictions. Consider the following simultaneous change in the values of the parameters:

$$\mu = 0.15,\ S'' = 4,\ \sigma_a = 0.001,$$

where σ_a is a new parameter, which controls costs associated with variable capital utilization. Thus, the capital that firms employ is the 'services' of capital, $u_t K_t$. Utilization of capital, u_t, generates adjustment costs in units of the final good that entrepreneurs must pay, in the amount $a(u_t) K_t$, where

$$a(u_t) = r^k \frac{\exp(\sigma_a (u_t - 1)) - 1}{\sigma_a}.$$

Here, r^k denotes the steady state rental rate of capital and σ_a is a parameter. The function, $a()$, is increasing and convex, and has value and slope zero at $u_t = 1$. Unity is the steady state value of u_t. The modification requires adjusting the rate of return on capital, R_t^k, and the resource constraint in obvious ways. The results are as follows. Note that now the economy without financial frictions responds much more strongly to a technology shock than does the economy without those frictions.

Percent Response in GDP to 1 percent Technology Shock

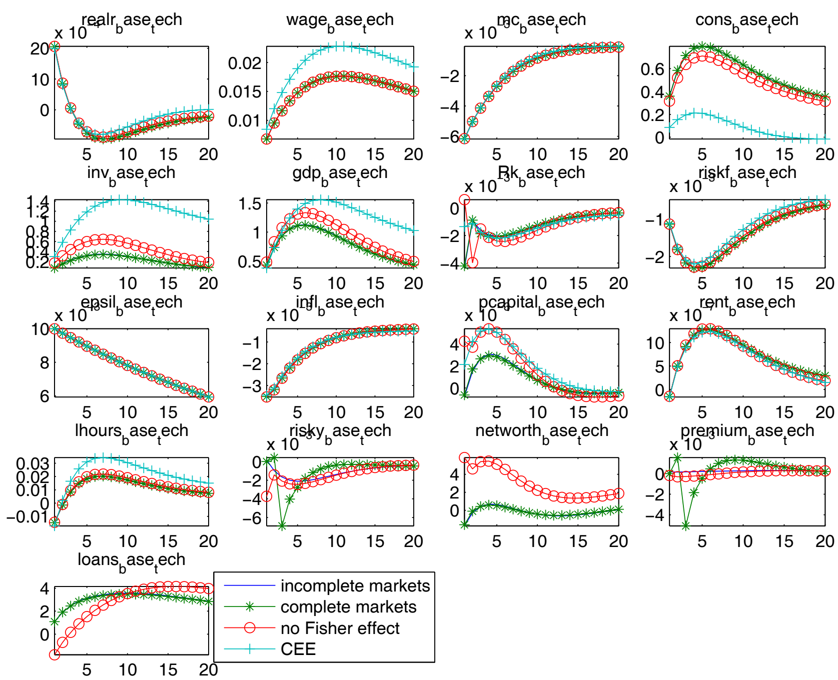
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	0.39	1.40	1.54	1.36	1.15	0.99	0.89
Incomplete	0.43	1.12	1.00	0.74	0.53	0.41	0.35
Incomplete/No Fisher	0.49	1.31	1.20	0.88	0.62	0.46	0.38
Complete	0.43	1.11	1.00	0.74	0.54	0.42	0.35

In the case of the monetary policy shock, output in the economy without frictions declines more than does output in the economy with the frictions.

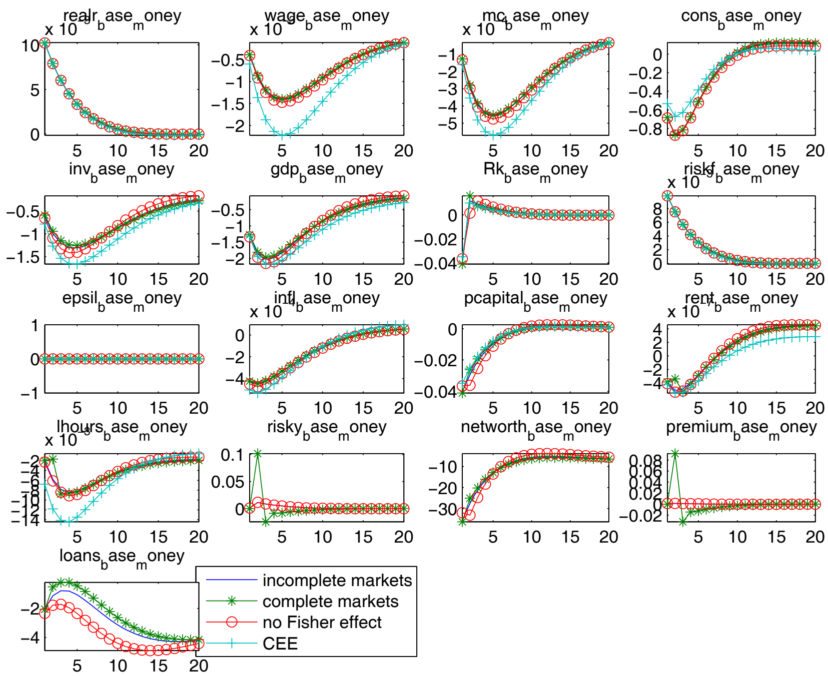
Percent Response in GDP to Monetary Policy Shock

	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	-1.32	-2.05	-1.28	-0.70	-0.40	-0.26	-0.20
Incomplete	-1.30	-1.82	-1.02	-0.48	-0.22	-0.12	-0.10
Incomplete/No Fisher	-1.34	-1.93	-1.02	-0.42	-0.15	-0.07	-0.06
Complete	-1.27	-1.77	-1.01	-0.50	-0.25	-0.14	-0.11

The impulse response functions are displayed as follows. The response to the technology shock is:



The responses to the monetary policy shock are:



7.2. Dropping the Non-Financial Frictions

Now consider the case in which there are no wage/price frictions (i.e., $\xi_p = \xi_w = 0$), monitoring costs are zero, there are no adjustment costs in capital ($S'' = 0$), and there is no habit persistence ($b = 0$). All other parameters are at their benchmark values listed above. In this case we obtain:

	Percent Response in GDP to Monetary Policy Shock						
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Incomplete	-12.76	-11.96	-11.20	-10.48	-9.79	-9.14	-8.53
Incomplete/No Fisher	1.49	0.05	0.05	0.04	0.04	0.04	0.03
Complete	-12.76	-11.96	-11.20	-10.48	-9.79	-9.14	-8.53

As expected, the monetary policy shock is neutral in the CEE model in this case. However, note how very non-neutral monetary policy is in the incomplete and complete markets models. This finding is consistent with the conclusions reached in section 3.3, because $\Theta > 0$. To further confirm those findings, we set $\Theta = 0$. In this case, equilibrium indeterminacy results if expected inflation appears in the Taylor rule, so we replaced it with current inflation (this is no surprise in light of the analysis in 3.3.) In this case, we obtained the following results

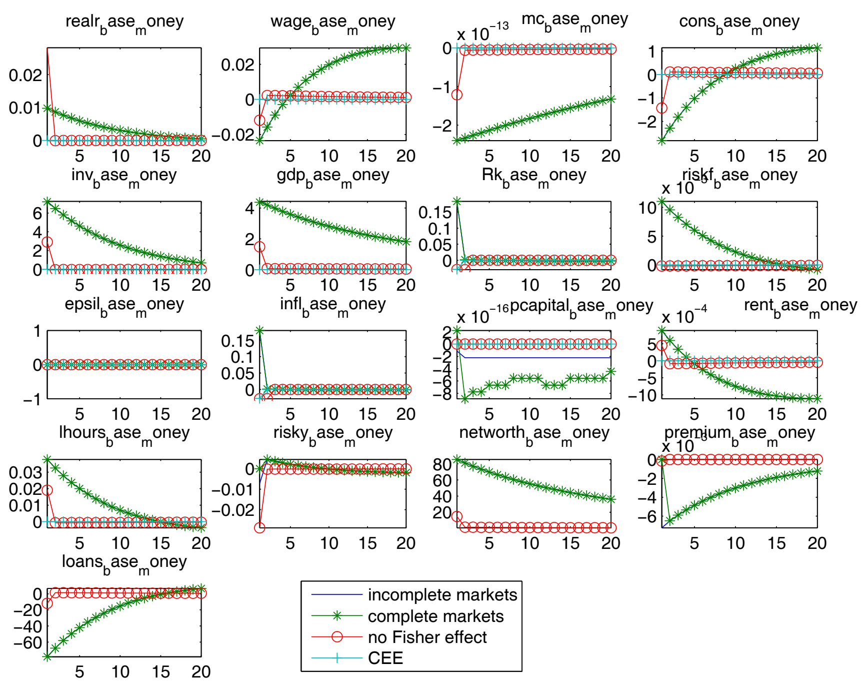
	Percent Response in GDP to Monetary Policy Shock						
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Incomplete	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Incomplete/No Fisher	0.79	0.03	0.02	0.01	0.01	0.01	0.01
Complete	0.00	0.00	0.00	0.00	0.00	0.00	0.00

This is as expected, although we do not have good intuition for the result in the no Fisher case.

We then reset $\Theta = 0.02$, and we also set $\mu = 0.10$. We then obtained the following very different results[4]:

	Percent Response in GDP to Monetary Policy Shock						
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Incomplete	4.44	3.59	2.95	2.45	2.05	1.74	1.48
Incomplete/No Fisher	1.48	0.06	0.05	0.05	0.04	0.04	0.03
Complete	4.38	3.59	2.94	2.45	2.05	1.74	1.48

This last result may at first seem odd. The positive monetary policy shock drives up the rate of interest and causes a jump in output. The jump is particularly pronounced when there is a Fisher effect. The following figure displays all the impulse responses. Note the enormous jump in net worth. In this model, the rise in net worth acts like a lump sum tax on ordinary households. They respond by working harder and consuming less. Investment jumps. These responses don't resemble the responses we think we see in the data.



We conclude that although the financial frictions are a source of non-neutrality, even in the absence of other frictions, they have counterfactual implications without those other frictions.

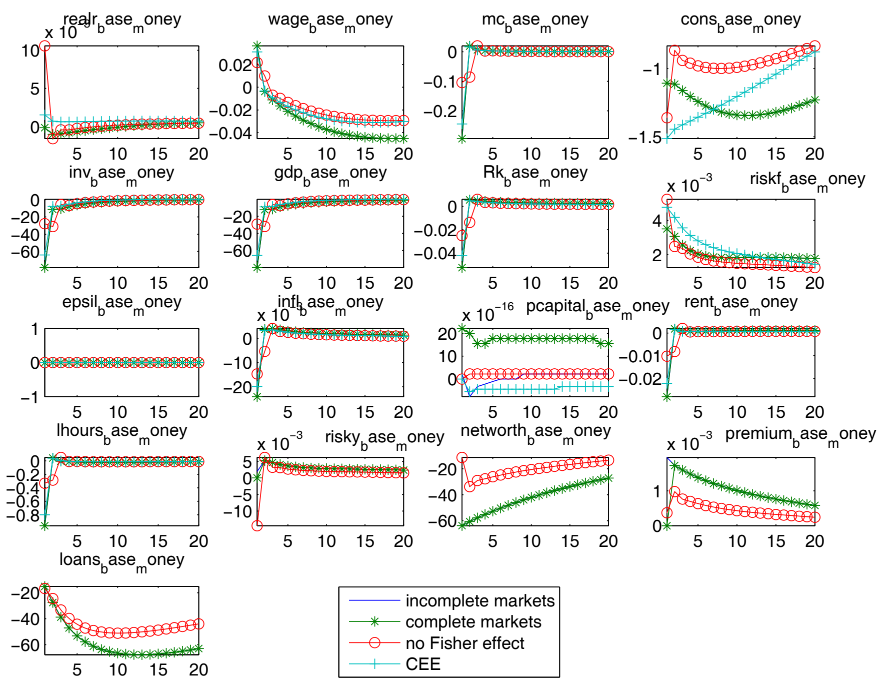
[4]In this case, the Taylor rule in the CEE model had current inflation (otherwise there would have been indeterminacy), but the Taylor rule in the other models was reset to having expected inflation one quarter in the future.

7.3. Adding Monetary Frictions But Leaving out Adjustment Costs and Habit

We now set ξ_p, ξ_w, μ and Θ at their benchmark values, and keep $S'' = b = 0$. Following are what happens after a monetary policy shock

Percent Response in GDP to Monetary Policy Shock							
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	-65.70	-4.87	-1.88	-0.90	-0.46	-0.23	-0.09
Incomplete	-79.91	-7.49	-3.82	-2.42	-1.68	-1.20	-0.86
Incomplete/No Fisher	-29.18	-5.54	-2.44	-1.41	-0.92	-0.63	-0.43
Complete	-79.91	-7.49	-3.82	-2.42	-1.68	-1.20	-0.86

Note how the fall in gdp is gigantic. This is true even in the CEE model.



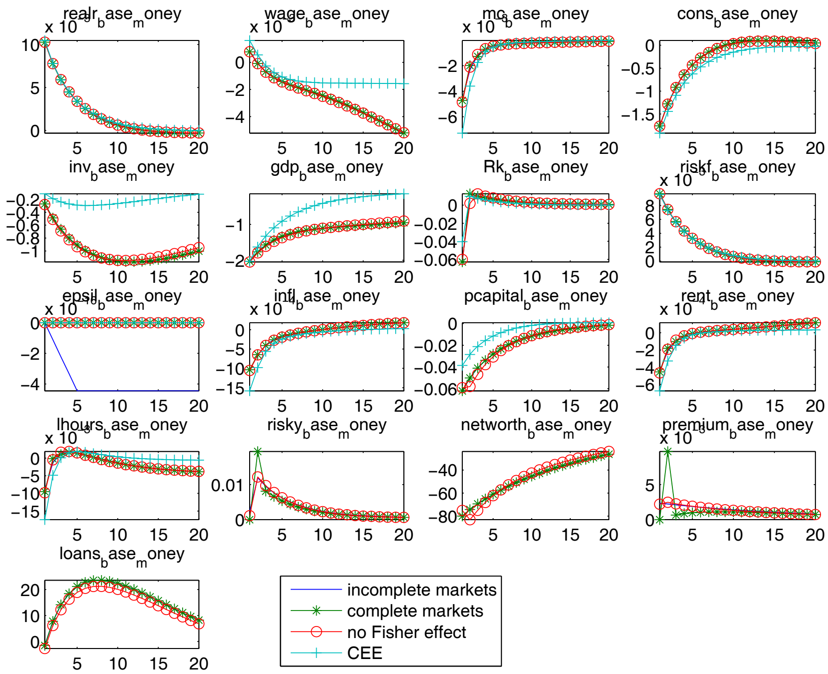
The jump in the interest rate creates a massive collapse in investment.

7.4. Adding Adjustment Costs and Habit

When we set S'' to its benchmark value, we obtain the following response to a monetary policy shock:

Percent Response in GDP to Monetary Policy Shock							
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	-2.03	-0.91	-0.48	-0.28	-0.19	-0.13	-0.10
Incomplete	-2.03	-1.34	-1.14	-1.06	-1.00	-0.93	-0.85
Incomplete/No Fisher	-2.05	-1.37	-1.14	-1.04	-0.97	-0.89	-0.80
Complete	-2.03	-1.34	-1.13	-1.06	-1.00	-0.93	-0.85

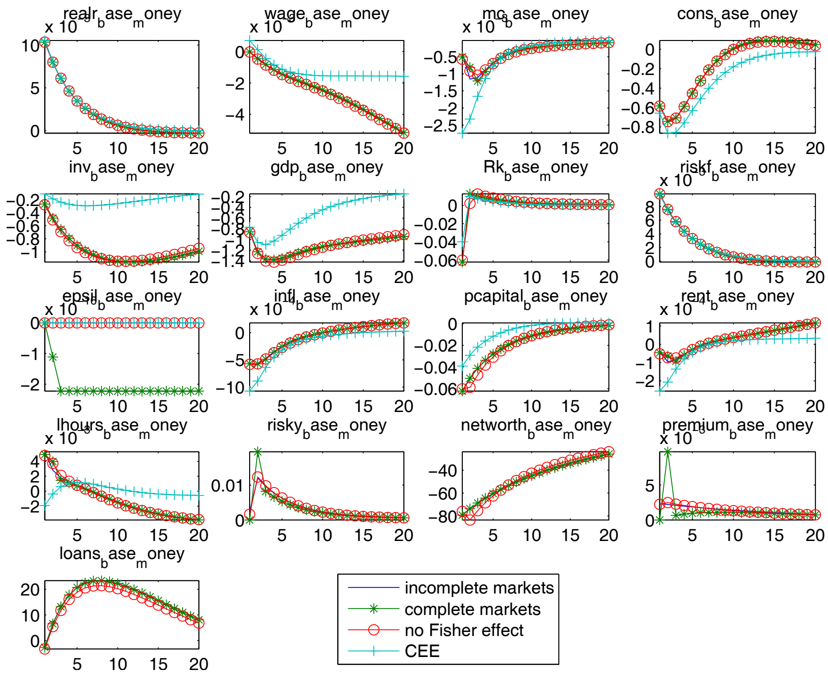
Note that now the response to a monetary policy shock is less gigantic. We can see this in the following figure too:



Next, we also set b at its benchmark value and obtained:

	Percent Response in GDP to Monetary Policy Shock						
	period 1	period 5	period 9	period 13	period 17	period 21	period 25
CEE	-0.77	-0.92	-0.52	-0.30	-0.19	-0.13	-0.10
Incomplete	-0.85	-1.35	-1.17	-1.07	-1.00	-0.93	-0.84
Incomplete/No Fisher	-0.87	-1.39	-1.19	-1.06	-0.97	-0.89	-0.80
Complete	-0.85	-1.35	-1.17	-1.06	-1.00	-0.92	-0.84

Note that now the fall in output is much smaller than it was in the absence of habit. The following set of impulse responses suggest that the reason lies in the smaller fall in consumption in the wake of a positive technology shock.



8. Steady State

To solve this model, we need to develop an algorithm for computing its steady state. In our analysis, we distinguish between steady state inflation, π, and the quantity appearing in the price and wage updating equations, $\bar{\pi}$. Equation (1.11) in steady state, is:

$$p^* = \left[\frac{(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\pi^{\iota_2}\bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right)^{\lambda_f}}{1-\xi_p\left(\frac{\pi^{\iota_2}\bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}} \right]^{\frac{1-\lambda_f}{\lambda_f}}.$$

Note that, if $\pi = \bar{\pi}$ then $p^* = 1$. Equation (1.12):

$$F_p = \frac{\lambda_z (p^*)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{k}{\mu_z}\right)^{\alpha} \left((w^*)^{\frac{\lambda_w}{\lambda_w - 1}} h \right)^{1-\alpha} - \phi \right]}{1 - \left(\frac{\pi^{\iota_2}\bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{1}{1-\lambda_f}} \beta \xi_p},$$

assuming

$$\left(\frac{\pi^{\iota_2}\bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{1}{1-\lambda_f}} \beta \xi_p < 1.$$

Equation (1.13) in steady state is:

$$F_p = \frac{\lambda_z \lambda_f \left(p^*\right)^{\frac{\lambda_f}{\lambda_f - 1}} \left[\left(\frac{k}{\mu_z}\right)^{\alpha} \left(\left(w^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} h\right)^{1-\alpha} - \phi\right] s}{\left[\frac{1-\xi_p \left(\frac{\pi^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{1-\lambda_f} \left[1 - \beta \xi_p \left(\frac{\pi^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]}$$

Equating the preceding two equations:

$$s = \frac{1}{\lambda_f} \frac{\left[\frac{1-\xi_p \left(\frac{\pi^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{1-\lambda_f} \left[1 - \beta \xi_p \left(\frac{\pi^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]}{1 - \left(\frac{\pi^{\iota_2} \bar{\pi}^{1-\iota_2}}{\pi}\right)^{\frac{1}{1-\lambda_f}} \beta \xi_p}. \tag{8.1}$$

In the case, $\pi = \bar{\pi}$, $s = 1/\lambda_f$. Equation (1.15) in steady state is:

$$F_w = \frac{\lambda_z \frac{\left(w^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} h}{\lambda_w}}{1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}},$$

as long as the condition,

$$\beta \xi_w \tilde{\pi}_w^{\frac{1}{1-\lambda_w}} \frac{\left(\frac{1}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}}}{\pi} < 1,$$

is satisfied. Also

$$\tilde{\pi}_w = \left(\pi\right)^{\iota_{w,2}} \bar{\pi}^{1-\iota_{w,2}}.$$

Equation (1.16) is

$$F_w = \frac{\left[\left(w^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} h\right]^{1+\sigma_L}}{\frac{1}{\psi_L} \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \tilde{w} \left[1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}\right]},$$

as long as

$$\beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} < 1.$$

Equating the two expressions for F_w, we obtain:

$$\tilde{w} = W \lambda_w \frac{\psi_L h^{\sigma_L}}{\lambda_z}, \tag{8.2}$$

where

$$W = \left(w^*\right)^{\frac{\lambda_w}{\lambda_w - 1}\sigma_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w}\right]^{\lambda_w(1+\sigma_L)-1} \frac{1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1 - \beta \xi_w \left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}}. \tag{8.3}$$

In steady state, (1.17) reduces to:

$$w^* = \left[\frac{(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w}}{1-\xi_w\left(\frac{\tilde{\pi}_w}{\pi}\right)^{\frac{\lambda_w}{1-\lambda_w}}} \right]^{\frac{1-\lambda_w}{\lambda_w}} \tag{8.4}$$

According to the wage equation, the wage is a markup, $W\lambda_w$, over the household's marginal cost. Note that the magnitude of the markup depends on the degree of wage distortions in the steady state. These will be important to the extent that $\tilde{\pi}_w \neq \pi_w$.

The marginal cost equation, (1.18) implies:

$$s = \frac{\tilde{w}}{(1-\alpha)}\left(\frac{\mu_z\,(w^*)^{\frac{\lambda_w}{\lambda_w-1}}\,h}{k}\right)^{\alpha},$$

where w^* is determined by (8.4). The steady state rental rate of capital is:

$$r^k = \alpha\left(\frac{\mu_z\,(w^*)^{\frac{\lambda_w}{\lambda_w-1}}\,h}{k}\right)^{1-\alpha} s. \tag{8.5}$$

In steady state, the capital accumulation equation, (1.20), is

$$\left[1-(1-\delta)\mu_z^{-1}\right]k = I.$$

In steady state, the equation for the nominal rate of interest, (1.21), reduces to:

$$1+R = \frac{\pi\mu_z}{\beta}. \tag{8.6}$$

In steady state, the marginal utility of consumption, (1.22), is

$$\lambda_z = \frac{1}{c}\frac{\mu_z - b\beta}{\mu_z - b}. \tag{8.7}$$

Finally, the euler equation for investment, (1.24), reduces to

$$q = 1.$$

We proceed as follows. First, fix the nominal rate of interest according to (8.6). Now, fix a value for r^k. Solve (8.5) for

$$\frac{h}{k} = \frac{1}{\mu_z}(w^*)^{\frac{\lambda_w}{1-\lambda_w}}\left(\frac{r^k}{\alpha s}\right)^{\frac{1}{1-\alpha}}, \tag{8.8}$$

where s is determined by (8.1). Then,

$$R^k = \left[r^k + (1-\delta)\right]\pi - 1.$$

Then, solve

$$[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}+\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega})-\mu G'(\bar{\omega})}\left[\frac{1+R^k}{1+R}\left(\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right)-1\right]=0.$$

for $\bar{\omega}$. Taking into account, $\Gamma' = 1 - F$ and $G' = \bar{\omega}F'$, we have

$$[1-\Gamma(\bar{\omega})]\frac{1+R^k}{1+R}+\frac{1-F(\bar{\omega})}{1-F(\bar{\omega})-\mu\bar{\omega}F'(\bar{\omega})}\left[\frac{1+R^k}{1+R}\left(\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right)-1\right]=0. \tag{8.9}$$

Next, find n/k which solves (2.3):

$$\frac{n}{k}=1-\frac{1+R^k}{1+R}\left[\Gamma(\bar{\omega})-\mu G(\bar{\omega})\right] \tag{8.10}$$

In steady state, (2.8) is

$$n=\frac{\gamma}{\pi\mu_z^*}\left\{R^k-R-\mu\int_0^{\bar{\omega}}\omega dF(\omega)\left(1+R^k\right)\right\}\left(\frac{k}{n}\right)n+w^e+\gamma\left(\frac{1+R}{\pi\mu_z^*}\right)n,$$

so that

$$\begin{aligned}
n &= \frac{w^e}{1-\frac{\gamma}{\pi\mu_z^*}\left\{R^k-R-\mu G\left(\bar{\omega}\right)\left(1+R^k\right)\right\}\left(\frac{k}{n}\right)-\gamma\left(\frac{1+R}{\pi\mu_z^*}\right)}, \\
k &= \left(\frac{k}{n}\right)n \\
h &= \left(\frac{h}{k}\right)k \\
I &= \left[1-(1-\delta)\mu_z^{-1}\right]k,
\end{aligned} \tag{8.11}$$

where $G\left(\bar{\omega}\right)$ is obtained from (2.4).

We now need to solve the resource constraint for consumption. But, first we require ϕ. We compute ϕ to guarantee that firm profits are zero in a steady state where $\pi = \bar{\pi}$. Let h^{ss} and k^{ss} denote hours worked and capital in such a steady state. Also, let F^{ss} denote gross output of the final good in that steady state. Write sales of final good firm as $F^{ss}-\phi$. Real marginal cost in this steady state is $s^{ss} = 1/\lambda_f$. Since this is a constant, the total costs of the firm are $s^{ss}F^{ss}$. Zero profits requires $s^{ss}F^{ss} = F^{ss} - \phi$. Thus, $\phi = (1 - s^{ss})\,F^{ss} = F^{ss}(1 - 1/\lambda_f)$, or,

$$\phi=\left(\frac{k^{ss}}{\mu_z^*}\right)^{\alpha}\left(h^{ss}\right)^{1-\alpha}\left(1-\frac{1}{\lambda_f}\right).$$

Solve the steady state version of the resource constraint, (2.10), for $c:$

$$d+c+I+\Theta\frac{1-\gamma}{\gamma}\left[n-w^e\right]=\left(p^*\right)^{\frac{\lambda_f}{\lambda_f-1}}\left(\frac{k}{\mu_z^*}\right)^{\alpha}\left[\left(w^*\right)^{\frac{\lambda_w}{\lambda_w-1}}h\right]^{1-\alpha}-\phi.$$

Compute the steady state real wage using (1.18):

$$\tilde{w}=s\left(1-\alpha\right)\left[\frac{\mu_z\left(w^*\right)^{\frac{\lambda_w}{\lambda_w-1}}h}{k}\right]^{-\alpha}. \tag{8.12}$$

Then, solve the labor supply equation, (8.2), for h :

$$h = \left[\frac{\lambda_z}{W\lambda_w\psi_L}\tilde{w}\right]^{\frac{1}{\sigma_L}},$$

where λ_z is obtained using (8.7). These calculations began by fixing a value for r^k. Adjust r^k until the value of h obtained from the above expression coincides with the value implied by (8.11).

It is of interest to understand what happens when $\mu = 0$. In this case, (8.9) implies $R = R^k$. So, one chooses r^k so that $R = \left[r^k + (1-\delta)\right]\pi - 1$. Then, (8.8) implies a value for h/k. From (8.11),

$$n = \frac{w^e}{1 - \frac{\gamma}{\beta}}.$$

In the case, $\bar{\pi} = \pi$, $\mu = 0$ implies:

$$\begin{aligned} c + I + \Theta\frac{1-\gamma}{\gamma}\left[n - w^e\right] &= \left(\frac{k}{\mu_z^*}\right)^\alpha h^{1-\alpha} - \left(\frac{k}{\mu_z^*}\right)^\alpha h^{1-\alpha}\left(1 - \frac{1}{\lambda_f}\right) \\ &= \frac{1}{\lambda_f}\left(\frac{1}{\mu_z^*}\right)^\alpha\left(\frac{h}{k}\right)^{1-\alpha} k, \end{aligned}$$

or,

$$\frac{c}{k} + \left[1 - (1-\delta)\mu_z^{-1}\right] + \Theta\frac{1-\gamma}{\gamma}\frac{\left[n - w^e\right]}{k} = \frac{1}{\lambda_f}\left(\frac{1}{\mu_z^*}\right)^\alpha\left(\frac{h}{k}\right)^{1-\alpha}$$

The labor-leisure choice implies:

$$c = \frac{\frac{\mu_z - b\beta}{\mu_z - b}}{W\lambda_w\psi_L}\tilde{w}h^{-\sigma_L},$$

where $\tilde{w}$ can be computed from (8.12) and $W = 1$ according to (8.3). Substituting this into the resource constraint, we obtain:

$$\frac{\frac{\mu_z - b\beta}{\mu_z - b}}{W\lambda_w\psi_L}\tilde{w}\frac{1}{h^{1+\sigma_L}} + \Theta\frac{(1-\gamma)\,w^e}{\beta - \gamma}\frac{1}{h} = \frac{\frac{1}{\lambda_f}\left(\frac{1}{\mu_z}\right)^\alpha\left(\frac{h}{k}\right)^{1-\alpha} - \left(1 - \frac{1-\delta}{\mu_z}\right)}{\frac{h}{k}},$$

which is a single equation in one unknown, h. Note that the right side must be positive for consumption to be positive. Also, the left side goes from 0 to ∞ as h goes from ∞ to 0. Thus, there is a unique solution, as long as the model implies positive steady state consumption. Once this is solved for h, then we have k. Then, given k we can compute $\bar{\omega}$ from (8.10):

$$\frac{n}{k} = 1 - \Gamma(\bar{\omega})$$

$$\Gamma(\bar{\omega}) = 1 - \frac{n}{k}$$

This gives the same solution as the model without financial frictions, except for the fact that entrepreneurs consume resources.