

Welfare Cost of Fluctuations when Labor Market Search Interacts with Financial Frictions

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Abstract

We provide a quantitative assessment of welfare costs of fluctuations in a search model with financial frictions. The matching process in the labor market leads positive shocks to reduce unemployment less than negative shocks increase it. We show that the magnitude of this non-linearity is magnified by financial frictions. This asymmetric effect of the business cycle leads to sizable welfare costs: financial frictions shift the distribution to regions with significantly higher and more asymmetric welfare costs. The model also accounts for the responsiveness of the job finding rate to the business cycle as financial frictions endogenously generate counter-cyclical opportunity costs of opening a vacancy and wage sluggishness.

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1 Introduction

Lucas (1987, 2003) shows that the welfare costs of business cycles are negligible. Since this provocative work, it seems that a large part of the literature does not challenge this result. This is surprising given the large literature on stabilizing and optimal policies using dynamic stochastic general equilibrium (DSGE) models¹: if welfare costs of the business cycle are negligible, how can we motivate the study of such policies? In this paper, we challenge Lucas's result by putting forward the view that a non-linear DSGE model, where the allocation is sub-optimal, can generate significant asymmetries at business cycle frequency: we show that the costs of recession can not be compensated by the gains of expansions. Therefore, average employment and average consumption are significantly lower than their deterministic steady-state levels. Welfare costs of fluctuations can be then significantly greater than those found by Lucas.² We argue that the interaction between labor market and financial frictions is essential to generate this result.

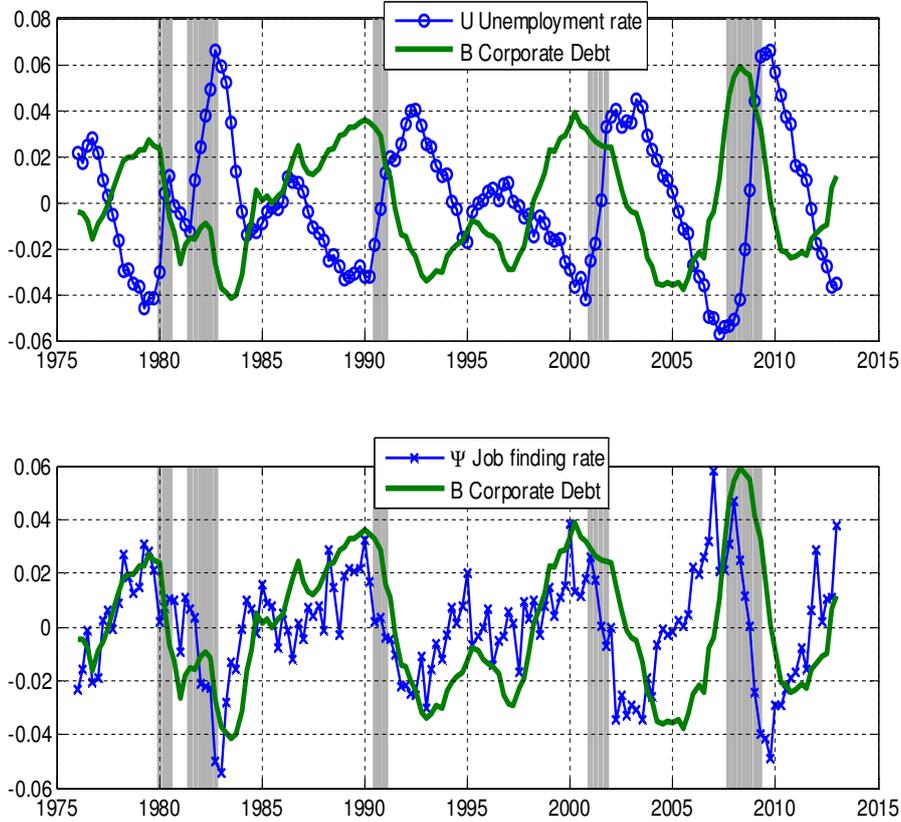
A quick look at the data confirms that financial and labor markets are intertwined. Figure 1 shows that starting from the mid- 1970s, recessions have been characterized both by deleveraging and increases in unemployment.³ It also shows that the episodes when jobs are created (the periods when the job finding rate rises) are also times when firms accumulate debt. Our paper aims at investigating how the interaction between financial and labor market frictions can generate large business cycle costs. We focus on the interplay between two non-linearities. The first non-linearity comes from the intrinsic structure of labor market frictions. Indeed, it takes time to search and form a new match, whereas the job separation is instantaneous. As a result, employment falls more during recessions than it increases during expansions. As put forward by Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013), canonical search-and-matching labor frictions therefore introduce a gap between the unemployment level at its deterministic steady state and its mean. They

¹In a standard New-Keynesian (NK) DSGE model, Woodford shows that the traditional arbitrage between inflation and output gap can be derived from individual welfare: thus, optimal policies based on this welfare criteria implicitly focus on welfare costs of the business cycle.

²If a significant gap between average and deterministic steady-state values exists, such costs are of a first order magnitude – as the costs of tax distortion also evaluated by Lucas (1987).

³Jermann & Quadrini (2012) also notice that debt repurchases (a reduction in outstanding debt) increase during or around recessions.

Figure 1: Cyclicality of unemployment rate U , job finding rate Ψ and debt stock B .



HP Filtered, logged, quarterly data. Recessions in shaded area (NBER dates). HP filtered data on U and Ψ have been divided by 4 for the purpose of scale consistency. Smoothing parameter 1600. Source : See Appendix A.

show how this generates sizable business-cycle costs.⁴ In this paper, we show that these non-linearities are amplified by the ones specific to financial frictions.⁵ Entrepreneurs must finance with debt the creation of news jobs. Credit market conditions therefore affect the cost of job creation. Tighter credit conditions (*i*) reduce the steady-state level of vacancies, making the economy more sensitive to the asymmetries of the labor market (level effect), (*ii*) generate a financial accelerator mechanism, which magnifies the impact of aggregate shocks and makes the economy more volatile (business cycle effect). Indeed, the imperfections of the credit market decrease the costs of the creation of new jobs after a boom – but less than

⁴In Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013), the computed welfare costs of the business cycle are between 0.2% and 1.2% of permanent consumption, which is between 4 and 24 times of the Lucas estimate of 0.05% (Hairault et al. (2010) calculate a welfare cost to be 0.55%, for Jung & Kuester (2011) this cost is 0.2%, and for Petrosky-Nadeau & Zhang (2013) it is 1.2%).

⁵Unlike Petrosky-Nadeau & Zhang (2013), we do not introduce any asymmetry in the process of the exogenous shocks. We thus focus on the endogenous asymmetries generated by the model.

they dampen hiring after a recession – thus generating a multiplier effect. Moreover, we show that the increase (decrease) in the credit costs in recession (expansion) leads to endogenous real wage rigidities.⁶ The endogenous wage rigidity magnifies the volatility of labor market aggregates, and thus the size of asymmetries.⁷ In summary, our model illustrates how financial frictions (*i*) enlarge the gap between unemployment (or the jobless rate) at the deterministic steady state and its mean, and (*ii*) allow to generate the magnitude of the observed fluctuation in a DSGE model with labor market search frictions.

More precisely, we introduce a DSGE model where labor markets are characterized by standard search-and-matching frictions à la Mortensen & Pissarides (1994) and where entrepreneurs' access to credit is limited by a collateral constraint because of enforcement limits, à la Kiyotaki & Moore (1997). We allow entrepreneurs to finance with debt also the intra-period costs associated to hiring. Credit costs associated to financial frictions lead to a lower equilibrium level for employment though an increase in labor costs (level effect). Moreover, we recover a credit multiplier mechanism (business-cycle effect) that significantly amplifies the propagation of productivity shocks.⁸

The quantitative computation confirms our intuition. Indeed, the introduction of financial frictions raises welfare costs of fluctuations several times with respect to the ones obtained with labor frictions only. The business cycle cost of fluctuations with financial and labor market frictions is 2.5% of workers' permanent consumption. It drastically falls without financial frictions and labor market frictions only (at 0.3% for workers). These costs are far larger than the estimates by Lucas (1987, 2003) who reports a welfare cost of 0.005% in the case of logarithmic utility. In addition, as in Petrosky-Nadeau & Zhang (2013), we compute the time-varying welfare cost and report its empirical distribution for the model with financial and search frictions and the model with search frictions only. Whatever the model, welfare costs are larger in recessions than welfare gains in expansion. This captures

⁶Since Shimer (2005)'s paper, real wage rigidity is known to be a possible solution to the volatility puzzle of the Diamond-Mortensen-Pissarides model.

⁷This differentiates us from Hairault et al. (2010) and Jung & Kuester (2011) or Petrosky-Nadeau & Zhang (2013), where real wages are exogenously fixed for the former, while for the latter, they are rigid via a bargaining à la Hall & Milgrom (2008).

⁸These two features (level and business-cycle effects) are shared with papers by Acemoglu (2001), Wasmer & Weil (2004), Petrosky-Nadeau (2013) and Petrosky-Nadeau & Wasmer (2013).

the asymmetric business cycles in our non-linear environment. In addition, the presence of financial frictions shifts the distribution of welfare costs to regions implying more asymmetry and greater losses: in presence of financial frictions, not only welfare costs are larger whatever the state of the economy (therefore welfare gains appear only in case of large expansions) but welfare gains remain small in case of expansion (while they increase quasi-linearly without financial frictions).

Our evaluation of welfare costs can be considered as reasonable only if based on a model able to match the main characteristics of the business cycle. From this positive perspective, our work refers to a recent literature of labor economics incorporating financial frictions into an environment characterized by labor market search. These works study how evolving conditions on credit markets affect the dynamics of labor markets and can improve the ability of standard search models to match the data. We show that the mechanisms that generate both the level and the business cycle effects on welfare can help solve the volatility puzzle emphasized by Shimer (2005). Indeed, as stressed by Shimer (2005), the textbook search and matching model cannot generate enough business-cycle-frequency fluctuations in unemployment, job vacancies, thus job finding rates (the "Shimer puzzle"). Indeed, in expansion, the increase in wages tends to lower the firms' incentive to hire more workers. In our model, in response to a positive productivity shock, counter-cyclical credit costs associated to financial frictions give firms an incentive to post more vacancies, independently from the expected benefit of new workers. Another channel through which financial and labor market frictions interact is the wage curve. As suggested by Chéron & Langot (2004) and Pissarides (2009), and in echo to old Keynes-Tarshis-Dunlop controversy, a solution of the puzzle must explain not only fluctuations in unemployment, but also in wages. We then show that financial frictions à la Kiyotaki & Moore (1997) allow us both to replicate volatilities of labor-market aggregates (worker flows and unemployment stock) and incorporate a calibration where a large part of the wage can fluctuate, as observed in the data. More precisely, we identify in the Nash bargaining process 3 counter-cyclical components: *i*) the counter-cyclical credit cost which directly affects the cost of vacancy posting; *ii*) credit conditions alter workers' and firms' stochastic discount factors along the business cycle. In boom, the decrease in credit costs leads entrepreneurs to be more patient. This naturally improves their relative

position during the wage bargaining process and thus dampens real-wage fluctuations; *iii*) in boom, the expected duration required to fill a vacancy goes up, which lowers the expected cost saving on vacancy posting, hence the surplus from a match. The impatient firm is more sensitive to this lower surplus than patient workers, which makes entrepreneurs reluctant to accept higher wages in economic booms. This allows the model to be consistent with the large changes in job finding rates observed in the data, while accounting for the cyclicity of real wages. In this respect, our work differs from Hagendorn & Manovskii (2008) who calibrate a small value of the worker bargaining power, targeting the regression coefficient between the HP-filtered log of wage and the HP-filtered log of productivity, but leading to "over-employment" at the equilibrium⁹ or Petrosky-Nadau & Wasmer (2013), Petrosky-Nadeau (2013) or Petrosky-Nadeau & Zhang (2013) who do not discuss the implications of their model with respect to the real wage dynamics. The stress on this endogenous wage rigidity – which is consistent with data – resulting from credit constraints constitutes the second contribution of our paper.

The paper is organized as follows. In section 2, we introduce the main mechanisms at work. Section 3 describes the model and section 4 focuses on equilibrium in the labor market. Our quantitative analysis is detailed in section 5. Section 6 concludes.

2 Welfare costs of fluctuations: why do search frictions and financial imperfections matter?

In this section, we introduce the main mechanisms at the roots of our results. Another way to understand why we obtain welfare costs of cycles, which are much greater than the ones resulting from Lucas (1987, 2003) is to focus on the fundamental difference between the two frameworks. In fact, while Lucas economy is "linear" (in the sense that the business cycle does not affect average unemployment and output), our model is characterized by non-

⁹We retain the restriction that the elasticity of the matching function with respect to unemployment is equal to the bargaining power of the workers. This calibration of worker's bargaining power does not a priori restricts the part of the wage that can fluctuate, as it is otherwise done in e.g. Hagendorn & Manovskii (2008). The "real wage rigidity" is thus an endogenous result in our case.

linearities due to the combination of labor market frictions and credit imperfections. In what follows, we show how non-linearities have important implications for welfare both via level effects, which are intrinsic in the structure of the model (section 2.1) and business cycle effects – i.e., business cycle fluctuations which are amplified by the financial accelerator (section 2.2). Notice that, with financial frictions, households can smooth their consumption using savings. We retain the simplifying assumption that saving is supplied by the households to finance entrepreneurs’ projects: land investments, as well as the costs associated with working capital within the period.

2.1 Non-linearities in the labor market

As in Lucas (1987, 2003), our exercise consists in a comparison between welfare in a stabilized economy (i.e., an economy that is always at the deterministic steady state) and welfare in a stochastic economy (i.e., its mean). As stressed by Hairault et al. (2010), non-linearities inherent to the standard search and matching model imply that the level of unemployment at its deterministic steady state is smaller than its mean. Due to congestion effects, average employment and therefore average consumption are lowered by the mere process of alternate expansions and contractions. Therefore, the welfare loss due to fluctuations in an economy characterized by labor frictions is not negligible. At the steady state, unemployment is then a convex function of the job finding rate Ψ :

$$\Psi U = sN \quad \Rightarrow \quad U = \frac{s}{s + \Psi}$$

with U the number of unemployed workers, s the exogenous job destruction rate and $N = 1 - U$, the number of employed workers, given that the population size is normalized to 1. Consider now a stochastic environment. Suppose for simplicity that Ψ ¹⁰ follows a Markov stochastic process defined over states i , and that unemployment converges instantaneously from one to another steady state, depending on the value of Ψ . Formally, conditional steady-

¹⁰We suppose for simplicity that the separation rate is constant. Indeed, Shimer (2012) shows that, since 1948, the job finding probability has been the main driving force behind fluctuations in the unemployment rate in the United States. Fluctuations in the employment exit probability have been quantitatively irrelevant during the last two decades.

state unemployment is $\tilde{u}_i = \frac{s}{s+\Psi_i}$. Moreover, stabilized unemployment is $\bar{u} = \frac{s}{s+\sum_i \pi_i \Psi_i}$ where π_i is the probability that Ψ takes the value Ψ_i and $\sum_i \pi_i \Psi_i$ is the mean of the job finding rate. Because of convexity,

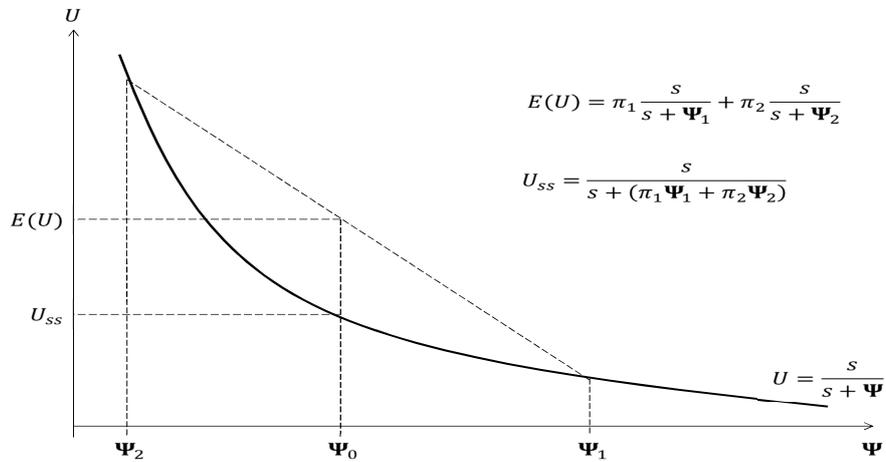
$$\frac{s}{s + \sum_i \pi_i \Psi_i} < \sum_i \pi_i \tilde{u}_i = \tilde{u} \approx E(u)$$

i.e., as can be seen from Figure 2, unemployment is a convex function of the job finding rate. Therefore, average unemployment is greater than the structural (stabilized) unemployment. Hairault et al. (2010) show that the unemployment gap is

$$\tilde{u} - \bar{u} \approx u''(\Psi) \frac{\sigma_\Psi^2}{2} \approx \frac{s}{(s + \Psi)^3} \sigma_\Psi^2 \quad (1)$$

which increases with σ_Ψ^2 and falls with Ψ .

Figure 2: Non linearities in the labor market: the level effect



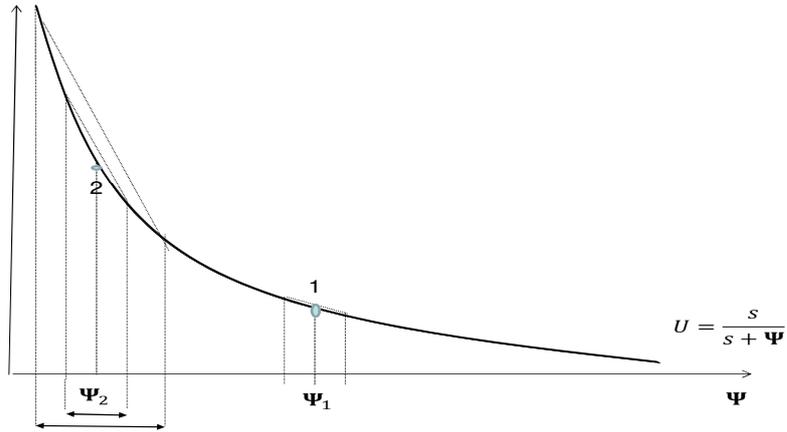
2.2 The credit multiplier throughout the cycle

As shown by equation (1), a mean-preserving increase in the volatility σ_Ψ widens the gap between the stabilized and fluctuating unemployment rates. The more volatile the job finding rate Ψ , the greater the business cycle cost. We thus expect large fluctuations to increase *per se* the unemployment gap.

Introducing financial frictions destabilizes the economy and triggers an amplification of cycles

(Kiyotaki & Moore (1997)). As the economy is more volatile, labor-market variables also experience greater fluctuations. This point is illustrated on Figure 3. As the volatility of the job finding rate Ψ increases, level effects associated to non-linearities are magnified. Figure 3 illustrates two economies: one characterized by labor frictions only (where $\Psi = \Psi_1$), and one with financial imperfections as well (where $\Psi = \Psi_2$). Because of financial frictions, the steady state of the job finding rate is lower (level effects), and the economy moves into the convex part of the function (see discussed in section 2.1). Moreover, because of the financial accelerator, the volatility of Ψ increases, therefore enlarging the unemployment gap.

Figure 3: The level effect is amplified by fluctuations in the job finding rate (business cycle effect)



3 The model

The economy is populated by two types of agents: firms and workers. The representative firm produces the final consumption good of the economy by combining labor and infrastructure (i.e., land). Firms have the possibility to finance their activity with loans funded by households. As debt contracts are not complete because of enforcement limits *à la* Kiyotaki & Moore (1997), firms are subject to a collateral constraint. Households can be either unemployed or employed workers by the firms. There is a canonical matching process *à la* Mortensen & Pissarides (1994) that allows firms to hire workers. Wages are set according to a standard Nash bargaining process.

In order to stress the economic mechanisms at work, we present a streamlined model without capital accumulation. Notice however that, even without capital, households can actually save by lending to firms. In addition, we lay stress on the extensive margin of labor, thereby discarding adjustments in hours, as in Blanchard & Gali (2010). Finally, we consider only technological shocks so as to be able to compare the model's welfare costs to those found in the literature.

3.1 Labor market flows

The economy is populated by a large number of identical households whose measure is normalized to one. Each household consists of a continuum of infinitely-lived agents. We consider a standard labor and matching model à la Mortensen & Pissarides (1994). Let N_t and M_t respectively denote the number of workers and the total number of new hires, and s the exogenous job destruction rate. Employment evolves according to:

$$N_t = (1 - s)N_{t-1} + M_t \tag{2}$$

where M_t , the number of hiring per period, is determined by a constant returns to scale matching function:

$$M_t = \chi V_t^\psi S_t^{1-\psi}, \quad 0 < \psi < 1$$

$\chi > 0$ is a scale parameter measuring the efficiency of the matching function, and V_t the number of vacancies. Following Blanchard & Gali (2010), we suppose that at the beginning of period t , there is a pool of jobless individuals, S_t , who are available for hire. This implies that the pool of jobless agents is larger than the number of unemployed workers. Indeed, at all times, individuals are either employed or willing to work (full participation) so that S_t is given by:

$$S_t = U_{t-1} + sN_{t-1} = 1 - (1 - s)N_{t-1} \tag{3}$$

where $U_t = 1 - N_t$ is the stock of unemployed workers when the size of the population is normalized to 1 and under the assumption of full participation. U_t measures thus the fraction of the population who are left without a job after hiring takes place in period t .

Among agents looking for a job at the beginning of period t , a number M_t are hired and start working in the same period. Only workers in the unemployment pool S_t at the beginning of the period can be hired ($M_t \leq S_t$). The ratio of aggregate hires to the unemployment pool $\Psi_t \equiv M_t/S_t$ is the rate at which the pool of jobless people find a job. Labor market tightness θ_t equals $\frac{V_t}{S_t}$.¹¹

3.2 Households

Households maximize an utility function of consumption and labor. Each period, an agent can engage in only one of 2 activities: working or enjoying leisure. There are employment lotteries to insure the individual idiosyncratic risk faced by each agent in his job match. Hence, the representative household's preferences are represented by:

$$E_0 \left[\sum_{t=0}^{\infty} \mu^t \{ N_t U^n(C_t^n) + (1 - N_t) U^u(C_t^u + \Gamma) \} \right] \quad (4)$$

where $0 < \mu < 1$ is the discount factor and Γ the utility of leisure. C_t^z stand for the consumption of employed ($z = n$) and unemployed agents ($z = u$). We assume

$$\begin{aligned} U(C_t^n) &= \frac{(C_t^n)^{1-\sigma}}{1-\sigma} \equiv \tilde{U}_t^n \quad \text{for employed workers} \\ U(C_t^u + \Gamma) &= \frac{(C_t^u + \Gamma)^{1-\sigma}}{1-\sigma} \equiv \tilde{U}_t^u \quad \text{for unemployed workers} \end{aligned}$$

with, $\sigma > 0$ the coefficient of relative risk aversion. With this utility function, our welfare measure indirectly depends on the employment level. Households labor opportunities evolves as follows:

$$N_t = (1 - s)N_{t-1} + \Psi_t S_t \quad (5)$$

Each household chooses $\{C_t^n, C_t^u, B_t\}$ to maximize (4) subject to the labor supply constraint (5) and the budget constraint

$$[N_t C_t^n + (1 - N_t) C_t^u] + B_t \leq R_{t-1} B_{t-1} + N_t w_t + (1 - N_t) b_t + T_t \quad (6)$$

¹¹The jobless rate is thus a convex function of the job finding rate, $S = \frac{s}{s+(1-s)\Psi}$.

where w is the real wage and b the unemployment benefit. T is a lump-sum transfer from the government. Moreover, B are private bonds financing firms and R is the gross investment return associated to these loans.¹²

3.3 Entrepreneurs and Firms

There are many identical firms in the economy. Entrepreneurs maximize the following sum of expected utilities:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^F) \right] \quad (7)$$

where β denotes entrepreneurs' discount factor, and $\mu > \beta$ implying that workers are more patient than firms¹³. Their budget constraint is:

$$C_t^F + R_{t-1}B_{t-1} + q_t[L_t - L_{t-1}] + w_tN_t + \bar{w}V_t \leq Y_t + B_t + \pi_t \quad (8)$$

where B is private debt, L productive land or infrastructure and q its price. Moreover, wN denotes total wages, with N the number of employees, whereas Y is the final output and π lump-sum dividends. Each firm has access to a Cobb-Douglas constant-return-to-scale production technology combining workers and infrastructure (land):

$$Y_t = A_t L_{t-1}^{1-\alpha} N_t^\alpha \quad (9)$$

where A_t represents the global productivity of factors in the economy, assumed to evolve stochastically as follows

$$\log A_t = \rho_a \log A_{t-1} + (1 - \rho_a) \log A + \varepsilon_t^a$$

with $\{\varepsilon^a\}_t$ the vector of iid innovations. Firms' activity can be financed by funds lent by households under imperfect debt contracts. Enforcement limits *à la* Kiyotaki & Moore

¹²See Appendix B for a complete description of the model.

¹³This assumption is needed to ensure that firms are debt constrained in equilibrium. We will discuss this point in the following.

(1997) imply that entrepreneurs are subject to collateral constraints. As Quadrini (2011) and Jermann & Quadrini (2012), we assume that, at the beginning of the period, firms can access financial markets to finance both their expenditures (i.e., entrepreneurs consumption and land investments) as well as the costs associated to the working capital within the period. Moreover, as Petrosky-Nadeau (2013) and Wasmer & Weil (2004), we suppose for simplicity that the costs associated to the working labor consist in hiring costs. Entrepreneurs can thus borrow from agents subject to the following collateral constraint

$$B_t + \omega V_t \leq m E_t [q_{t+1} L_t] \quad (10)$$

where m is the (exogenous) loan to debt ratio. The firms' constraint associated to the evolution of vacancies is:

$$N_t = (1 - s)N_{t-1} + \Phi_t V_t \quad (11)$$

where $\Phi_t \equiv M_t/V_t$ is the rate at which a vacancy is filled, considered as exogenous by the firm.

Each entrepreneur chooses $\{C_t^F, L_t, B_t, V_t, N_t\}$ to maximize (7) subject to (8) where Y_t is given by (9), (10) and (11). We denote respectively λ_t^F , $\lambda_t^F \varphi_t$, and ξ_t the Lagrange multipliers associated to (8), (10) and (11).

3.4 Wage

Wages are determined via a generalized Nash bargaining between individual workers and firms, ie.:

$$\max_{w_t} \left(\frac{\mathcal{V}_t^F}{\lambda_t^F} \right)^\epsilon \left(\frac{\mathcal{V}_t^H}{\lambda_t} \right)^{1-\epsilon} \quad (12)$$

with $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}}$ the marginal value of a match for a firm and $\mathcal{V}_t^H = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}}$ the marginal household's surplus from an established employment relationship. ϵ denotes the firm's share

of a job's value, i.e., firms' bargaining power. The wage curve is

$$\begin{aligned}
w_t = & \underbrace{\epsilon b}_{(a)} + \underbrace{(1 - \epsilon) \frac{\partial Y_t}{\partial N_t}}_{(b)} \\
& + (1 - \epsilon)(1 - s)\beta E_t \left[\underbrace{(1 + \varphi_{t+1}) \frac{\lambda_{t+1}^F}{\lambda_t^F}}_{(3)} \underbrace{\left\{ \frac{\bar{\omega}}{\Phi_{t+1}} \underbrace{\frac{\beta \frac{\lambda_{t+1}^F}{\lambda_t^F} - \mu \frac{\lambda_{t+1}}{\lambda_t}}{\beta \frac{\lambda_{t+1}^F}{\lambda_t^F}}}_{(1)} + \underbrace{\frac{\mu \frac{\lambda_{t+1}}{\lambda_t}}{\beta \frac{\lambda_{t+1}^F}{\lambda_t^F}} \bar{\omega} \theta_{t+1}}_{(2)} \right\}}_{(\Sigma)} \right] \quad (13)
\end{aligned}$$

where (a) represents the weight of the reservation wage in total wage and (b) + (Σ) is the workers' gain from the match. This gain can be decomposed into the marginal productivity of the new employed worker, (b), and the saving on search costs if the job is not destroyed next period (Σ).¹⁴

In presence of discounting heterogeneity, the bargaining process is influenced by the fact that workers and firms evaluate the aggregate surplus differently. As firms are less patient than workers, they under-estimate the value of the intertemporal surplus. In practice, the value of the agreement after a meeting is for them lower than for workers. Thus, term (1) in (Σ) reduces the bargained wage of an amount proportional to the gap between the firm price kernel and the one of the workers. This allows wages to account for the surplus gap between workers and firm – associated to the gap in impatience rates.

In addition, during the bargaining process, the firm-worker pair shares the returns on the search process. For the worker, this is equal to the discounted time duration to find a job offer; for the firm, returns are instead equivalent to the discounted time duration to find a worker. Note however that these relative time spans cannot be proxied by the ratio of the average duration for these two search processes – as it would be the case without discounting heterogeneity. Given that firms are less patient than workers, the “subjective” duration of the search process is as well under-estimated by the firm. Thus, term (2) of (Σ) pushes

¹⁴When firms and workers have the same discount factor, (13) collapses to the standard Blanchard & Gali (2010) wage curve. Note that by eliminating financial frictions (let $\varphi = 0$, $\mu = \beta$, $\lambda_t = \lambda_t^F$) we can recover the standard wage rule in Blanchard & Gali (2010): $w_t = (1 - \epsilon) \left(\frac{\partial Y_t}{\partial N_t} + \bar{\omega}(1 - s)\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \theta_{t+1} \right] \right) + \epsilon b$.

up the wage through the ratio of discount rates. Indeed, entrepreneurs are impatient: this pushes them to quicker negotiations and to accept paying higher wages. Note that this second mechanism has been emphasized by Rubinstein (1982): once entered a bargaining process, the most impatient agent eventually gains less from the match.¹⁵ Moreover, the outcome of the two mechanisms is amplified by the "credit channel", term (3), $(1 + \varphi_{t+1})$, which is also at work in Petrosky-Nadeau (2013) and Wasmer & Weil (2004) and discussed in Monacelli et al. (2011).

3.5 Markets clearing

In order to close the model, we assume that the government does not accumulate debt and pays unemployment benefits by the means of the revenues collected with the lump-sum tax, i.e.: $T_t = (1 - N_t)b_t$. It is possible to show (see Becker (1982), Becker & Foias (1982)) that in presence of standard levels of uncertainty¹⁶, firms are collateral constrained at each period. Thus, the debt limit eventually determines the equilibrium level of corporate debt and workers savings. The private-bonds market is thus cleared. Good market equilibrium requires:

$$Y_t = N_t C_t^n + (1 - N_t) C_t^u + \bar{\omega} V_t + C_t^F$$

Finally, as discussed above, we assume that land supply is fixed and that the land market clears at each period, i.e.: $L_t = 1$.

4 On the interactions between labor and financial frictions

In what follows, we analyze in the details the mechanisms at the roots of the the level effect (section 4.1) and the business cycle effect (section 4.2), shown in Figure 3, respectively.

¹⁵Notice however that, in Rubinstein (1982), the effect of term (1) is not present because of the eventually static approach of the game.

¹⁶For some discussion see, among others, Iacoviello (2005).

4.1 The labor market at the steady state

The first effect of the financial friction is the fall in the incentive to open vacancies, thereby lowering the job finding rate Ψ . This shifts the equilibrium on the more convex part of the relation between the unemployment and the job finding rate. This first section is devoted to discuss this steady state interaction between labor and financial frictions.

4.1.1 Wage curve WC

Using the equation (13) and the FOC of the household and entrepreneur problems¹⁷, given that Φ is a function of θ , the wage curve (WC) can be rewritten as:

$$w = \epsilon b + (1 - \epsilon) \frac{\partial Y}{\partial N} + (1 - s)(1 - \epsilon) \bar{\omega} \mu \left(1 + \underbrace{1 - \frac{\beta}{\mu}}_a \right) \left[\underbrace{\frac{\theta^{1-\psi}}{\chi} \left(\frac{\beta - \mu}{\mu} \right)}_b + \underbrace{\theta}_c \right] \quad (14)$$

At the steady state, the "credit multiplier" is summarized by relative impatience. The relative impatience has two contrasting effects on WC : on the one hand, workers know that the continuing match will allow the firm to save money on future hiring costs (all the more so, as hiring costs are borrowed from patient households). As a result, workers want a share of this future gain for the firm: the larger the impatience gap (i.e. term a in (14)), the greater the wage. On the other hand, firms' gain in absence of separation is dampened by the fact that impatience reduces the intertemporal surplus of the firm, and this pushes down wages. This is represented by term b in (14) (which interacts with c).

In absence of discounting heterogeneity (i.e. term $a = b = 0$) we recover the standard Blanchard & Gali (2010)'s steady-state wage curve:

$$w = \epsilon b + (1 - \epsilon) \frac{\partial Y}{\partial N} + (1 - s)(1 - \epsilon) \bar{\omega} \mu \theta \quad (15)$$

By comparing (15) with (14), we see that the WC of our model is steeper or flatter with respect to the standard Pissarides-Blanchard & Gali (2010) baseline depending on the net

¹⁷More precisely the equations equations (19) and (23) in appendices B.1 and B.2.

effect of terms a , b and c . The slope of the WC of our framework is:

$$Slope_{WC} = \left. \frac{\partial w}{\partial \theta} \right|_{WC} = (1-s)(1-\epsilon)\bar{\omega} \left(1 + 1 - \frac{\beta}{\mu} \right) \left[\frac{(1-\psi)}{\theta^\psi \chi} (\beta - \mu) + \mu \right]$$

that is positive when $\frac{(1-\psi)}{\theta^\psi \chi} \left(1 - \frac{\beta}{\mu} \right) < 1$.¹⁸ Indeed, the larger the impatience gap, the greater the "credit multiplier". For realistic impatience gaps and our standard calibration of the labor market, this condition is always verified.

Let now focus on the effects of an increase in relative impatience. Keeping μ fixed and close to one, an increase in the heterogeneity gap entails a lower β . In this case

$$\frac{\partial Slope_{WC}}{-\partial \beta} = -(1-s)(1-\epsilon)\bar{\omega} \left[\frac{(1-\psi)}{\theta^\psi \chi} \left(3 - \frac{2\beta}{\mu} \right) - 1 \right]$$

when the relative impatience increases (i.e. β decreases), the WC becomes more sloped when $\frac{(1-\psi)}{\theta^\psi \chi} \left(3 - 2\frac{\beta}{\mu} \right) > 1$. When the WC is upward sloped, this condition is always verified for any impatience gap. Thus, a decrease in β makes the WC steeper (see Figure 4).

Note finally that relative impatience does not affect the intercept of the curve, equal to a weighted average of the outside opportunity $O \equiv b$ and marginal product of labor $MPL \equiv \frac{\partial Y}{\partial N}$.

4.1.2 Job creation curve JC

Consider now the job creation (JC) curve at the steady state:¹⁹

$$w = \frac{\partial Y}{\partial N} + \theta^{1-\psi} \frac{\bar{\omega}}{\chi} \left(1 + 1 - \frac{\beta}{\mu} \right) [(1-s)\beta - 1] \quad (16)$$

¹⁸More precisely, $(1-\psi)$ and $\beta - \mu$, in absolute values, are less than 1 while $\frac{1}{\theta^\psi} > 1$ and the value of χ is determined at the steady state given the calibration on labor market facts. As at the steady state we have $\chi = \Phi \theta^{1-\psi} < 1$ for a calibrated Φ , $\frac{1}{\chi} > 1$. In a nutshell, this condition is very likely to hold at the steady state but we still need to check it using the benchmark calibration

¹⁹We use equation (17) of the entrepreneur problem in appendix B.2 and $\varphi = \left(1 - \frac{\beta}{\mu} \right)$, from equations (19) and (23) in appendices B.1 and B.2, taken at the steady state.

The slope of JC is:

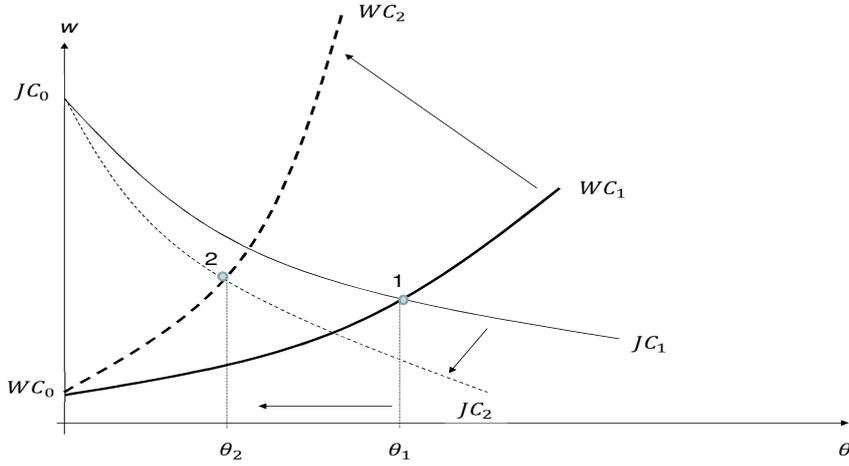
$$Slope_{JC} = \left. \frac{\partial w}{\partial \theta} \right|_{JC} = (1 - \psi) \theta^{-\psi} \frac{\bar{w}}{\chi} \left(1 + 1 - \frac{\beta}{\mu} \right) [(1 - s) \beta - 1] < 0$$

JC is unambiguously downward sloping since $(1 - s) < 1$ and $\beta < 1$. The increase in relative impatience entices the firm to post less vacancies. In addition, with a higher gap in impatience rates, the firm becomes more patient, and value more the future gain from the match. An increase in relative impatience can be proxied as a decrease in β , i.e.:

$$\frac{\partial |Slope_{JC}|}{-\partial \beta} = (1 - \psi) \theta^{-\psi} \frac{\bar{w}}{\chi} \left[\frac{1}{\mu} + (1 - s) 2 \left(1 - \frac{\beta}{\mu} \right) \right] > 0$$

It is straightforward to see that greater impatience gaps entail a steeper curve. The intercept is not affected. Notice also that relative impatience makes the JC curve unambiguously steeper than the standard JC curve (see Figure 4).²⁰

Figure 4: Labor market equilibrium with increasing discounting heterogeneity



²⁰In absence of discounting heterogeneity, we recover the standard JC :

$$w = \frac{\partial Y_t}{\partial N_t} + \theta^{1-\psi} \frac{\bar{w}}{\chi} [(1 - s) \mu - 1]$$

4.1.3 Steady state equilibrium

The above analysis has shown the implications of discounting heterogeneity for both the wage and job creation curves, – and thus, for the labor market equilibrium. Under the restriction that the WC has a positive slope, an increase of the discounting gap, measuring a rise in the financial frictions, makes both WC and JC steeper. Therefore, the steady-state level of market tightness is smaller than the baseline search and matching model. Figure 4 illustrates how the labor market equilibrium is affected by increasing heterogeneity in discount factors.

Proposition 1. *The steady-state level of market tightness in presence of discounting heterogeneity is smaller than in the baseline search and matching model.*

Proof. See Appendix B.4. This result gives theoretical foundations for the shift to the left of Ψ in Figure 3.

4.2 The impact of the business cycle

We now analyze the impact of a standard productivity shock in presence of credit and labor frictions: how do financial frictions affect the mechanisms at the heart of labor market dynamics? In what follows, we discuss the impact of a technological shock on the job creation curve (JC) and the wage curve (WC), respectively.

4.2.1 The Job Creation curve JC .

We now focus on the (JC) condition and on the impact of financial frictions on the opportunity cost of opening a vacancy. The (JC) curve is:

$$\bar{\omega} \frac{(1 + \varphi_t)}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t + (1 - s) \beta E_t \left[\frac{\lambda_{t+1}^F}{\lambda_t^F} \frac{\bar{\omega}}{\Phi_{t+1}} (1 + \varphi_{t+1}) \right] \quad (17)$$

Term φ_t in equation (17) is the Lagrangian multiplier associated to the credit constraint and represents the "credit multiplier" of this model. It appears both on the LHS and RHS of equation (17). Nevertheless, there is almost no persistence in the adjustment of φ_t : after

a jump at the time of the shock, it comes back to its steady state value. We thus shift our attention on its impact on the LHS of (17). In recession, tight credit conditions (large values of φ_t) drive up the opportunity costs associated to vacancy posting, $\bar{\omega}(1 + \varphi_t)$. This introduces a counter-cyclical and time-varying wedge that has the potential to magnify the productivity shocks. If the real wage is sufficiently rigid, the adjustments of quantities can thus be large on the labor market. In Petrosky-Nadeau (2013), the interaction between the labor market demand and the financial frictions mechanism is different. The credit multiplier *à la* Bernanke et al. (1999) is indeed associated to the productivity threshold insuring firms to default and it is amplified throughout the cycle by a time-varying and counter-cyclical monitoring cost carried by banks. In fact, for the mechanism associated to canonical financial frictions *à la* Bernanke et al. (1999) to match data, an ad-hoc counter-cyclical monitoring cost is incorporated into the financial contract.²¹

²¹Note finally that eliminating the vacancy-posting cost from the debt limit is not sufficient to rule out financial frictions from the model. Indeed, entrepreneurs are still constrained in their ability to borrow, even if they do not borrow to finance the working capital. We recover in this case a standard collateral constraint *à la* Kiyotaki & Moore (1997), where firms borrow to finance current economic activity (land purchases, debt service), including entrepreneurs consumption. In order to recover the frictionless Blanchard & Gali (2010) standard case, we need to eliminate agents impatience heterogeneity ($\beta = \mu$, and thus $\lambda_t^F = \lambda_t$) and let $\varphi_t = 0$. However, in this case:

$$\frac{\bar{\omega}}{\Phi_t} = \frac{\partial Y_t}{\partial N_t} - w_t + (1 - s)\beta E_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\bar{\omega}}{\Phi_{t+1}} \right]$$

By solving this equation, it follows that the rate at which labor is hired depends on the expected discounted stream of marginal profits generated by an additional hire.

4.2.2 The wage curve WC .

The wage response to the business cycle can be analyzed by using the following log linear approximation. Let $y = \frac{\partial Y}{\partial N}$, then:

$$\begin{aligned} \hat{w}_t &= \frac{(1-\epsilon)y}{b+(1-\epsilon)(y-b+\Sigma)} \hat{y}_t + \frac{(1-\epsilon)\Sigma}{b+(1-\epsilon)(y-b+\Sigma)} \hat{\Sigma}_t \\ \text{with } \hat{\Sigma}_t &= \underbrace{\frac{\varphi}{1+\varphi} E_t \hat{\varphi}_{t+1}}_{(3)} + \underbrace{\left(E_t \hat{\lambda}_{t+1}^F - \hat{\lambda}_t^F \right)}_{(4)} - \underbrace{\frac{\frac{\beta-\mu}{\beta} \bar{\omega}}{\frac{\bar{\omega}}{\Phi} \frac{\beta-\mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \hat{\Phi}_{t+1}}_{(1a)} \\ &\quad - \underbrace{\frac{\frac{\bar{\omega}}{\Phi} \frac{\mu}{\beta}}{\frac{\bar{\omega}}{\Phi} \frac{\beta-\mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \left(E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{\lambda}_{t+1}^F + \hat{\lambda}_t^F \right)}_{(1b)} \\ &\quad + \underbrace{\frac{\bar{\omega} \theta \frac{\mu}{\beta}}{\frac{\bar{\omega}}{\Phi} \frac{\beta-\mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \hat{\theta}_{t+1}}_{(2a)} + \underbrace{\frac{\bar{\omega} \theta \frac{\mu}{\beta}}{\frac{\bar{\omega}}{\Phi} \frac{\beta-\mu}{\beta} + \bar{\omega} \theta \frac{\mu}{\beta}} \left(E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{\lambda}_{t+1}^F + \hat{\lambda}_t^F \right)}_{(2b)} \end{aligned}$$

where hat variables denote log deviations from the steady state. In response to a productivity shock, as in the standard matching model: i) constant unemployment benefits (term (a) in (13)) lead wage to be a-cyclical whereas, ii) when ϵ is not close to one, productivity (terms (b) in (13)) and the labor market tightness (θ_{t+1} in Σ) lead wages to be pro-cyclical.²² When the search value (Σ) matters, financial interactions also matter because they are also in Σ . One can identify three effects of financial frictions:

- *Fluctuations in relative stochastic discount rates.* These fluctuations are given by $E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{\lambda}_{t+1}^F + \hat{\lambda}_t^F$. Equilibrium conditions imply that this term varies in the same direction as $\hat{\varphi}_t$, which is counter-cyclical.²³ In booms, the credit constraint

²²Notice that Hagendorn & Manovskii (2008)'s calibration consists in generating a rigid wage by imposing ϵ close to 1: this ensures that labor demand is barely affected by changes in wages. In addition, the large value for b chosen by these authors magnifies the impact of the technological shocks on labor demand.

²³We use the following approximation of the Euler equations on consumption:

$$\begin{aligned} \left. \begin{aligned} R_t \mu E_t [\lambda_{t+1}] &= \lambda_t \\ R_t \beta E_t [\lambda_{t+1}^F] &= (1-\varphi_t) \lambda_t^F \end{aligned} \right\} \approx \left\{ \begin{aligned} \hat{R}_t + E_t [\hat{\lambda}_{t+1}] &= \hat{\lambda}_t \\ \hat{R}_t + E_t [\hat{\lambda}_{t+1}^F] &= -\frac{\varphi}{1-\varphi} \hat{\varphi}_t + \hat{\lambda}_t^F \end{aligned} \right. \\ \Rightarrow E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - E_t \hat{\lambda}_{t+1}^F + \hat{\lambda}_t^F &= \frac{\varphi}{1-\varphi} \hat{\varphi}_t \end{aligned}$$

represents for firms a smaller financial burden than at the steady state ($\widehat{\varphi}_t < 0$) and the gap in worker's and firm's discount factors falls. Term (1b) shows that the underestimation of the match surplus by the firm decreases, implying an upward pressure on wages. However, the same force acts in the opposite direction through term (2b): entrepreneurs are more patient in expansions, leading them to delay negotiations in order to pay lower wages. The total impact of heterogenous discounting dynamics on wages is counter-cyclical, because $\frac{\bar{\omega}}{\Phi} \frac{\mu}{\beta} - \bar{\omega} \theta \frac{\mu}{\beta} > 0$.²⁴

- *Fluctuations in the time duration to fill a vacancy.* In expansion, the time duration to find another worker increases. (the probability of filling a vacancy falls: $\widehat{\Phi}_{t+1} < 0$ in (1a)). Thus, the gain from the match (i.e., the cost saved on future vacancy posting when the worker and the firm match today) falls. Given that firms are relatively short-sighted ($\beta < \mu$), this point is particularly important to them, which makes them reluctant to accept higher wages. This component is also a counter-cyclical force in the wage equation.
- *Fluctuations in the credit costs.* In boom, the costs to fill a vacancy are reduced through lower credit costs (term (3)), entailing wage moderation. This counter-cyclical component of the bargained wage is also present in Petrosky-Nadeau (2013).

Hence, the financial frictions introduce different counter-cyclical components in the wages. This contributes to magnify the volatility of quantities in the labor market, without introducing a strictly rigid wage. One contribution of our paper is to show that the standard search model with credit frictions à la Kiyotaki & Moore (1997) can both replicate the observed volatility of the labor-market quantities and wage.²⁵ This dimension is neglected in Petrosky-Nadeau (2013) and in Petrosky-Nadeau & Zhang (2013); analogously, results reported in Petrosky-Nadeau & Wasmer (2013) suggest that their type of financial frictions does not have any significant impact on the wage elasticity to aggregate shocks.

²⁴Indeed, we have $sign\left(\frac{\bar{\omega}\theta\frac{\mu}{\beta} - \frac{\bar{\omega}}{\Phi}\frac{\mu}{\beta}}{\bar{\omega}\theta\frac{\mu}{\beta} - \frac{\bar{\omega}}{\Phi}\frac{\mu}{\beta}}\right) = sign\left(\theta - \frac{1}{\Phi}\right) = sign\left(\frac{\Psi-1}{\Psi}\right) = -$. We reduce the analysis to the numerator because the denominator is positive, around $\beta \approx \mu$ and $\mu \approx 1$: $\frac{\bar{\omega}}{\Phi} \frac{\beta-\mu}{\beta} + \bar{\omega}\theta \frac{\mu}{\beta} = \frac{\bar{\omega}\theta}{\beta\Psi} (\beta - \mu + \mu\Psi) > 0$.

²⁵Even if the regression coefficient between the HP-filtered log of wage and the HP-filtered log of productivity is well reproduced in Hagendorn & Manovskii (2008)'s model, it is not shown in the paper that the volatility of the real wage is matched.

5 Quantitative analysis

After presenting the calibration, we first show that the model is able to match the magnitude of the business cycle. As the relevance of our quantitative calculation of business cycle costs depends on our ability to match the volatility of data, overcoming the "Shimer puzzle" is a necessary condition for our exercise. We then measure the welfare costs of business cycle.

5.1 Calibration

The calibration is based on quarterly US data.²⁶

Preference and technology parameters : The discount factor for patient agents is consistent with a 4% annual real interest rate. For the impatient consumer, we set $\beta = 0.99$, which corresponds to a value in the range of the ones chosen by Iacoviello (2005)²⁷. The risk aversion is set to 1 for firms and 2 for workers. Both values lie within a standard interval in the literature. In addition, the firm is characterized by a lower risk aversion because, as shown by Iacoviello (2005), such a calibration ensures that, for a wide range of values for volatility shocks, impatience and loan-to-value ratio m , the borrowing constraint is binding.

Financial parameters: The corporate debt-to-GDP ratio pins down the value of m in the collateral constraint. We use as target the average 2001-2009 of corporate debt over GDP (debt outstanding, annual data, corporate sector, Flow of Funds Accounts tables of the Federal Reserve Board).

Labor market parameters: Employment level N is consistent with Hall (2005)'s estimates of the average unemployment rate ($N = 0.88$). According to Hall (2005), observed high transition rate from out of the labor force directly to employment suggests that some fraction of those classified as out of the labor force are nonetheless effectively job-seekers.

²⁶The data are described in appendix A.

²⁷Notice that it is sufficient to have a very small degree of impatience heterogeneity for debt limits to hold

Hall (2005) adjusts the US unemployment rate to include individuals who are out of the labor force who are actually looking for a job.

As in Shimer (2005), the quarterly separation rate s is 0.10, so jobs last for about 2.5 years on average. Using steady-state labor-market flows, we infer Ψ given s and N . This leads to $\Psi = 0.423$. This value is lower than in the usual Mortensen-Pissarides model. Indeed, the pool of job seekers is larger in Blanchard & Gali (2010) than in the standard MP model.

The elasticity of the matching function with respect to the number of job seekers is $\psi = 0.5$, which lies within the range estimated in Petrongolo & Pissarides (2001). This value is also chosen so as to illustrate the pure effect of the non-linearities in the unemployment dynamics in the model without financial frictions (see section 2.1), as opposed to the non-linearities in the job finding rate (see Hairault et al. (2010)). The efficiency of matching, χ , is set such that firms with a vacancy find a worker with a 95% probability within a quarter, which is consistent with Andolfatto (1996).

As stressed by Hagendorn & Manovskii (2008), the parameters that determine the responsiveness of job creation to business cycles are the ratio of unemployment benefits (or home production without policy) to the wage and the firm's bargaining power. The utility of leisure parameter, Γ , is pinned down so as to match an unemployment benefit equal to 0.7 at steady state. This gives $b/w = 0.720$ (consistently with Hall & Milgrom (2008)). The cost of posting a vacancy, ω , is set to 0.17 as in Barron & Bishop (1985) and Barron et al. (1997). We obtain $\frac{\omega V}{Y} = 0.0179$, which is in the range found in the literature (0.005 in Chéron & Langot (2004), 0.01 in Langot (1995) or 0.05 in Krause & Lubik (2007)). Notice finally that, in order to reproduce the volatility of the job finding rate, Hagendorn & Manovskii (2008) need to calibrate $b/w = 0.955$ and $\epsilon = 0.9480$, which implies that the share of wage that can fluctuate is negligible. In this paper, with $b/w = 0.720$ and $\epsilon = 0.5$, half of the wage can fluctuate. Thus, there is room for economic mechanisms to endogenously lead wages to fluctuate (which is consistent with the data). In addition, the model must endogenously generate limited fluctuations in w so as to preserve firms' incentives to hire in booms.

Shocks: The technological shock is calibrated as in Hairault et al. (2010). We choose the standard deviation of technological shock to reproduce the observed GDP standard deviation. Table 1 summarizes the calibration. ²⁸

Table 1: Calibration

(a) External information			
Notation	Label	value	Reference
β	discount factor (impatient)	0.99	Iacoviello (2005)
α	production function	0.99	Iacoviello (2005)
σ_W	risk aversion, worker	2	
σ_F	risk aversion, firm	1	Iacoviello (2005)
s	Job separation rate	0.1	Shimer (2005)
N	Employment	0.88	Hall (2005)
ψ	Elasticity of the matching function	0.5	Petrongolo & Pissarides (2001)
ξ_V	financial constraint	1	Petrosky-Nadeau (2013)
ω	cost of job posting	0.17	Barron et al. (1997) and Barron & Bishop (1985)
$\frac{b}{w}$	replacement ratio	0.72	Hall and Milgrom (2008)
A	average TFP	1	Normalization
ρ_A	Persistence	0.95	Hairault et al. (2010)
(b) Empirical target			
Notation	Label	value	Empirical target
μ	discount factor (patient)	$1/(1.04^{\frac{1}{4}})$	Annual real rate of 0.04
χ	scale parameter of matching function	0.63397	Probability of filling a vacancy $\Phi = 0.95$
m	collateral constraint	0.61	corporate debt to GDP ratio $B/Y = 0.595$
σ_A	Standard deviation	0.0046	Observed σ_Y
(c) Derived parameter values			
Notation	Label	value	
Ψ	Job finding rate	0.423	
Γ	preference	0.19	

5.2 Business cycle properties

In this section, we document the unconditional business cycles facts on financial variables and labor market adjustments. Our contribution lies in bringing together financial data (from Jermann & Quadrini (2012)) and data from the labor market literature (Shimer (2012)). We also lay stress, in both markets, on fluctuations in quantity (debt, unemployment) as

²⁸We check that, for the benchmark calibration, the conditions on the slopes and steepness of WC hold: $\frac{(1-\psi)}{\theta^\psi \chi} \left(1 - \frac{\beta}{\mu}\right) < 1$ and $\frac{(1-\psi)}{\theta^\psi \chi} \left(3 - 2\frac{\beta}{\mu}\right) > 1$.

well as equilibrium price (interest rate, wage).²⁹ Table 2, column 1, reports business cycle properties found in the data.

Table 2: Business cycle volatility : Model versus data

	1		2	
	Data		Model	
	std(.)		std(.)	
Y	1.44	**	1.43	**
C	0.81	*	0.92	*
N	0.72	*	0.59	*
Y/N	0.54	*	0.42	*
w	0.63	*	0.68	*
U	7.90	*	4.26	*
Ψ	5.46	*	4.35	*
V	9.96		8.51	
B	1.68	*	1.52	*
R	0.92	*	0.25	*
$corr(U, \Psi)$	-0.91		-0.86	
$corr(U, V)$	-0.97		-0.71	

** std (in percentage); * relative to GDP std

The volatility of real wages is not close to zero. Moreover, this volatility is larger than the one of labor productivity. This clearly suggests that real wage rigidity (implying a zero standard deviation for fluctuations in w) is not a realistic explanation for the strong cyclicity of labor market aggregates. $corr(U_t, V_t)$ summarizes the dynamics around the Beveridge curve. The negative covariance is consistent with the view that productivity shocks have a more important weight than reallocation shocks at business cycle frequency: any increase in job vacancies lead to more hirings, hence lower unemployment. Moreover, $corr(U_t, \Psi_t)$ captures the relationship between the probability of finding a job and business cycle changes in unemployment. As expected, it is negative. Jung & Kuester (2011) point out that mean unemployment exceeds steady-state unemployment when the job-finding rate and the unemployment rate are non-positively correlated and the average job-finding rate is lower than the steady-state job-finding rate.³⁰

²⁹All data have been re-computed and updated so that our sample covers 5 recession episodes from 1976 January through January 2013 (see Appendix A for a complete description of the data set). In our view, previous papers who study the interaction between financial and real variables in DSGE models (Monacelli et al. (2011), Christiano et al. (2010)) summarize labor market adjustments using only fluctuations in employment and unemployment.

³⁰This can be inferred from the employment-flow equation taken at the steady state $sN_t = \Psi U_t$ where

The second moments from simulated data are reported in Table 2, column 2. Comparing columns 1 and 2 of Table 2, it can be seen that the model performs quite well in matching volatilities of employment and vacancies. Moreover, for what concerns financial variables, the model also matches the volatility of corporate debt.

Simulations confirm that financial frictions actually generate wage stickiness. Indeed, at business cycle frequencies, the decomposition of the wage equation confirms that counter-cyclical components dominate pro-cyclical elements. As a result, the wage increase in expansion is dampened. Firms have then a stronger incentive to create jobs, which raises the job finding rate. The wage relative standard deviation is actually a little higher than the one found in the data. This can account for the fact that the model's job finding rate is a slightly less volatile than in the data. This suggests that the endogenous wage sluggishness in the model is consistent with the wage dynamics in the data. This allows the model to generate sufficiently large movements in job finding rates. Our model solves the Shimer' volatility puzzle without introducing the counterfactual assumption of a constant real wage.

5.3 Welfare cost of fluctuations

5.3.1 Decomposing the welfare cost of fluctuations: level effect and business cycle effect

The expected lifetime utility of a worker is:

$$\tilde{U}^w = E_0 \sum_{t=0}^{\infty} \mu^t [N_t U(C_t^m) + (1 - N_t) U(C_t^u + \Gamma)] = E_0 \sum_{t=0}^{\infty} \mu^t U(C_t + (1 - N_t)\Gamma)$$

because $C_t \equiv N_t C_t^m + (1 - N_t) C_t^u$ and the FOC on consumption imply $C_t^m = C_t^u + \Gamma$. How much steady state-consumption would workers be ready to give up to be indifferent between the steady state and the fluctuating economy? This welfare cost of fluctuations τ is such

$N_t = 1 - U_t$. Hence, $sE(1_t - U_t) = cov(U_t, \Psi_t) + E(U_t)E(\Psi_t)$. Subtracting the steady-state from both sides of the above equation, leads to $E(U_t) - u_t = -\frac{1}{s+\Psi} [cov(U_t, \Psi_t) + (E(\Psi_t) - \Psi_t)E(U_t)]$. We deduce that if (i) $E(\Psi_t) - \Psi_t < 0$ and (ii) $cov(U_t, \Psi_t) < 0$, then necessarily, $E(U_t) - u_t > 0$. The correlation at the bottom of Table 2 suggests that (ii) holds in the data.

that

$$\sum_{t=0}^{\infty} \mu^t [\bar{N}U(\bar{C}^n(1-\tau)) + (1-\bar{N})U((\bar{C}^u + \Gamma)(1-\tau))] = \sum_{t=0}^{\infty} \mu^t U([\bar{C} + (1-\bar{N})\Gamma](1-\tau)) = \tilde{U}^w$$

where variables denoted with an overbar are set at their steady state values.³¹ We deduce:

$$\tau = 1 - \left[\tilde{U}^w \frac{(1-\mu)(1-\sigma)}{(\bar{C} + (1-\bar{N})\Gamma)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}$$

The result is reported in Table 3, line 1, column A. The business cycle cost of fluctuations with financial frictions is 2.5 % of workers' permanent consumption. This number is far larger than the estimates found by Lucas (1987, 2003) who reports a welfare cost of $\tau = 0.005\%$ with log utility. It is noticeable that the welfare costs are large even though agents can save by lending to firms: workers can actually smooth business cycles with savings.

A way to understand these quantitative results is to decompose the business cycle cost. We use a Taylor expansion of welfare in the volatile economies. The crucial point at this stage is to consider the Taylor expansion around the mean of the stochastic economy, and not around the deterministic steady state. This ensures that the computation of the welfare of the stochastic economy takes into account the gap between the mean of the stochastic steady state and the deterministic steady state. This leads to approximate the difference in welfare in stabilized and volatile economies as the sum of a level effect and the business cycle effect. Indeed, we have:

$$\tilde{U}^w \approx \frac{1}{1-\mu} U(E_0[C + (1-N)\Gamma]) \left[1 - \frac{1}{2} \sigma(1-\sigma) (\gamma_c \text{Var}(\hat{c}) + \gamma_u \text{Var}(\hat{u}) + \gamma_{cu} \text{Cov}(\hat{c}, \hat{u})) \right]$$

where we denote³² $\hat{x} = \frac{X_t - E_0[X]}{E_0[X]}$, for $x = C, U$ and $\gamma_c = \frac{E_0[C^2]}{E_0[(C+(1-N)\Gamma)^2]}$, $\gamma_u = \frac{\Gamma^2 E_0[(1-N)^2]}{E_0[(C+(1-N)\Gamma)^2]}$

³¹A similar computation can be done for the firm's owner:

$$\sum_{t=0}^{\infty} \beta^t U(\bar{C}^F(1-\tau^F)) = \tilde{U}^F = \sum_{t=0}^{\infty} \beta^t U(C_t^F)$$

When introducing financial frictions, firms' welfare cost of fluctuations can also be taken into account. In this case, aggregate welfare costs would be then greater. We choose to focus on worker's welfare cost only so as to compare our results to the existing literature.

³²Indeed, given that $\frac{\partial U}{\partial C} = U'$, $\frac{\partial U}{\partial N} = -\Gamma U'$, $\frac{\partial^2 U}{\partial C^2} = U''$, $\frac{\partial^2 U}{\partial N^2} = \Gamma^2 U''$ and $\frac{\partial^2 U}{\partial C \partial N} = -\Gamma U''$, we obtain, with

and $\gamma_{cu} = \frac{2\Gamma E_0[C(1-N)]}{E_0[(C+(1-N)\Gamma)^2]}$. This leads to

$$(1 - \tau) \approx \left(\frac{E_0[C + (1 - N)\Gamma]}{\bar{C} + (1 - \bar{N})\Gamma} \right) \left[1 - \frac{1}{2}\sigma(1 - \sigma) (\gamma_c Var(\hat{c}) + \gamma_u Var(\hat{u}) + \gamma_{cu} Cov(\hat{c}, \hat{u})) \right]^{\frac{1}{1-\sigma}}$$

If we neglect the level effect, then we have $U(E_0[C + (1 - N)\Gamma]) \approx U(\bar{C} + (1 - \bar{N})\Gamma)$ because we assume that $E_0[C + (1 - N)\Gamma] \approx \bar{C} + (1 - \bar{N})\Gamma$, then

$$1 - \tau_{BC} = \left[1 - \frac{1}{2}\sigma(1 - \sigma) (\gamma_c Var(\hat{c}) + \gamma_u Var(\hat{u}) + \gamma_{cu} Cov(\hat{c}, \hat{u})) \right]^{\frac{1}{1-\sigma}}$$

where τ_{BC} denotes the welfare costs of the BC computed in the spirit of Lucas (1987, 2003). In contrast, if we neglect the business cycle effect ($\gamma_c Var(\hat{c}) + \gamma_u Var(\hat{u}) + \gamma_{cu} Cov(\hat{c}, \hat{u}) \approx 0$), we have

$$(1 - \tau_L) = \frac{E_0[C + (1 - N)\Gamma]}{\bar{C} + (1 - \bar{N})\Gamma}$$

where τ_L denotes the welfare costs of the business cycle effect linked to the level effect. We deduce that $(1 - \tau) = (1 - \tau_{BC})(1 - \tau_L) \Rightarrow \tau \approx \tau_{BC} + \tau_L$. Numerical computations give τ and τ_L given $E_0[C]$ and \bar{C} , $E_0[U]$ and \bar{U} . The previous formula then gives τ_{BC} .³³

Table 3: Decomposition of welfare costs of business cycle

	Worker with financial frictions A	Worker without financial frictions B
Total welfare cost		
1. $\tau \times 100$	2.50	0.30
Decomposing the welfare cost		
2. $\tau_L \times 100$	2.45	0.24
3. $\tau_{BC} \times 100$	0.05	0.06
4. $E_0[C + (1 - N)\Gamma^u]/(\bar{C} + (1 - \bar{N})\Gamma^u)$	0.975	0.997
5. $\sqrt{Var(\hat{c})} \times 100$	2.59	2.85
6. $\sqrt{Var(\hat{u})} \times 100$	7.23	0.12
7. $Cov(\hat{c}, \hat{u})$	-0.19	-0.34

line 1 = line 2 + line 3

Welfare costs due to business cycle fluctuations alone, τ_{BC} , are reported in Table 3, line 3,

the usual functional form $U(x) = \frac{x^{1-\sigma}}{1-\sigma}$, $U' = x^{-\sigma}$ and $U'' = -\sigma x^{-\sigma-1} = -\sigma(1 - \sigma) \frac{x^{1-\sigma}}{1-\sigma} \frac{1}{x^2}$.

³³Same considerations apply for firms' owners, using $E_0[C^F]$, \bar{C}^F and τ^F .

column A, and equal 0.05% of permanent consumption for workers. The most interesting result is the measure of $\tau_L = 2.45\%$ in Table 3, line 2, column A. It accounts for the large increase in business cycle cost compared with Lucas' estimate. Indeed, in Lucas (1987, 2003), $\tau_L = 0$. By assumption, there is no gap between average and steady state consumption. Our model shows that this approximation is not acceptable because business cycle volatility significantly affects the gap between average and steady state employment and consumption. Thus, the business-cycle costs are sizable: they are 50 times greater than in Lucas' estimate. If we take our computations to the data, the contrast with Lucas results is straightforward. Our calculations show that, with financial frictions, the mean of employment is $E(N) = 0.85123$, while its steady state value is $\bar{N} = 0.88$. This implies that the level effect associated to employment only is $100 \times \frac{\bar{N} - E(N)}{\bar{N}} = 3.3$. If we apply this percentage to US civilian employment stock in 2013, the job losses associated to fluctuations represent more than 4 millions jobs. Analogous considerations apply for GDP per capita. The comparison between $E(Y)$ and \bar{Y} lead to the following estimate: the level effect entails a GDP per capita loss of 1569.3 dollars a year.

Without financial frictions, the welfare cost for workers falls drastically ($100 \times \tau = 0.3$, line 1, column B, table 3).³⁴ Without financial constraints, the magnitude of the business cycle costs is reduced to 6 times Lucas' evaluation. Our numerical computations show that the level effect entail a loss of about 300.000 jobs and each household loses 104.03 dollars per year.

5.3.2 Asymmetric welfare costs of business cycles

As stressed by Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013), search and matching models predict asymmetric responses to business cycle shocks as recessions (expansions) are characterized by severe and rapid rises (gradual decline) in unem-

³⁴This estimate is computed by using our model, once we eliminate discounting heterogeneity and collateral constraints, but keeping the parameter values reported in Table 1 (panels (b) and (c)). The rationale behind this approach is the following. The model with financial frictions is considered as the "true" model of the economy. By calibrating the model to match key financial and labor market targets, we uncover the "true" parameter values. The business cycle costs without financial frictions are then computed using these parameter values.

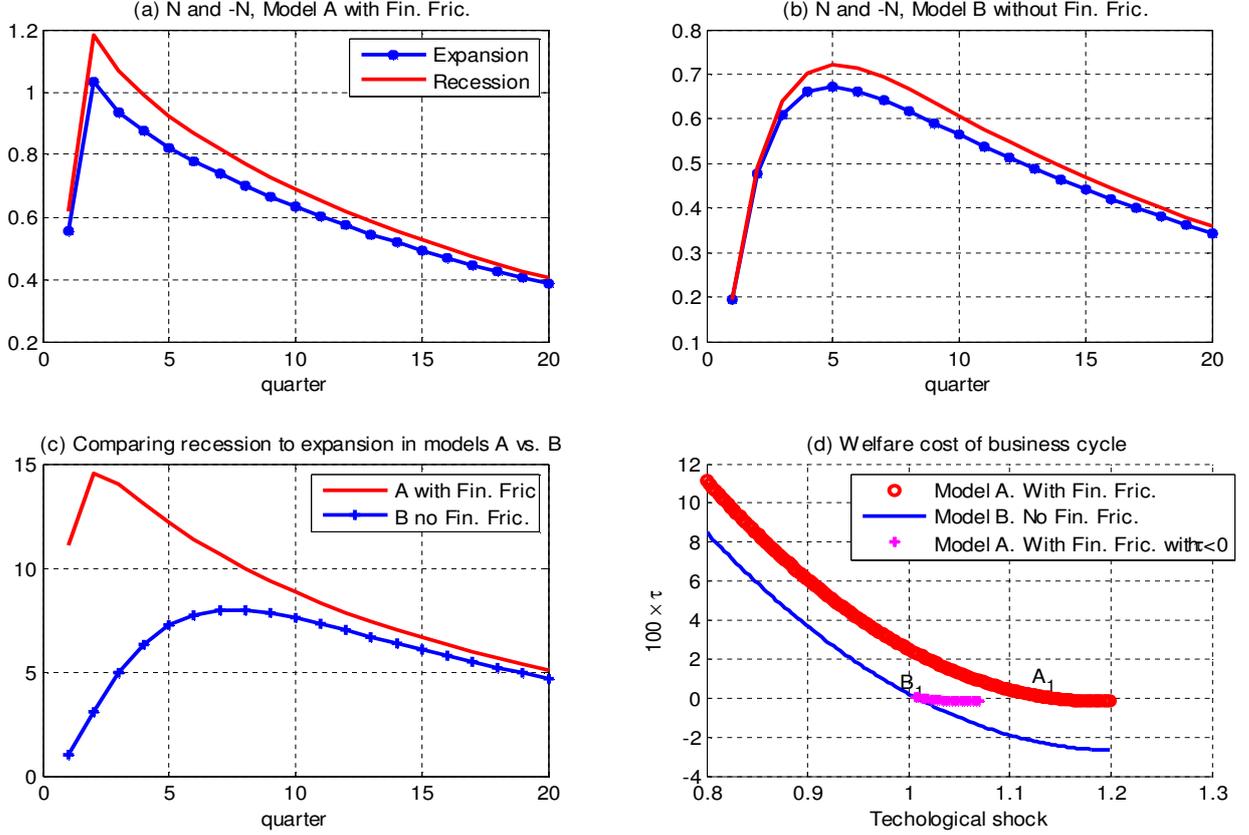
ployment. These asymmetric fluctuations are consistent with empirical evidence (MacKay & Reis (2008), Petrosky-Nadeau & Zhang (2013) among others). The IRFs in model B (without financial frictions) are consistent with these features. Figure 5, panel (b) displays the employment response to a one-standard-deviation increase in technology (dotted line) as well as the employment response in case of a negative shock of the same magnitude (solid line). Employment falls more in recession than it increases in boom. We report in Figure 5, panel (a), the IRFs from model A (with financial frictions) in recession versus economic boom. The asymmetric response of employment is even larger than in model B. In order to measure this increased asymmetry due to the presence of financial frictions, we report, in panel (c), the gaps between IRFs in recession vs. boom in both models. The gap between IRFs is twice larger in the economy with financial frictions (model A) than in a economy without financial frictions (model B): the maximum of the gap is around 15% in model A, whereas it is only 7.5% in model B.³⁵ In addition, since labor market adjustments are connected to credit market conditions, the immediate response of credit market conditions directly affect employment dynamics. The maximum of the gap in model A is reached after 2 quarters, whereas it is reached after 7 quarters in model B.

In order to measure the implied business cycle costs of these non-linearities, we compute the time-varying welfare cost τ as in Petrosky-Nadeau & Zhang (2013) for model A (with financial frictions) and model B (without financial frictions)³⁶. Figure 5, panel (d), plots the welfare cost $\tau \times 100$ against the technological shock. First, in model B, the welfare cost is counter-cyclical. In addition, welfare gains in expansion are much lower than welfare costs in recession, which is consistent with the asymmetric responses to business cycle shocks displayed in matching models: starting from point B_1 , a 10% increase in productivity (from 1 to 1.1) results in a welfare gain of 2% while, for a fall in productivity of the same magnitude (from 1 to .9), the welfare loss is twice larger. Interestingly, welfare costs in model A are

³⁵This last case summarizes the results reported in Hairault et al. (2010), Jung & Kuester (2011) and Petrosky-Nadeau & Zhang (2013).

³⁶The welfare cost at each date is based on the comparison between the deterministic worker's welfare and the fluctuating worker's welfare summarized by his value function. This value function is time-varying. Its expected value is computed, for each current state of the economy, using the 2nd-order Dynare approximation of endogenous variables. We pay attention to consider decision rules around the mean, rather than the steady state, which is the default setting in Dynare

Figure 5: The Welfare Cost of Business Cycles and Asymmetry: Why do financial frictions matter.



(a): IRF of Employment $\times 100$, N in expansion (dotted line) and $-N$ in recession (solid line) in model A with financial frictions. one-standard-deviation shock. deviation from steady state. (b): IRF of Employment $\times 100$, N in expansion (dotted line) and $-N$ in recession (solid line) in model B without financial frictions. one-standard-deviation shock. deviation from steady state. (c) : $\frac{-IRF_{recession} - IRF_{expansion}}{IRF_{expansion}} \times 100$ in models A (solid line) and B (dotted line). (d) Steady state value of technological shock is 1. Curves are quadratic fit over 30,000 simulations for model A (circle), model B (solid line). + : segment of cost function for model A (only segment with $\tau < 0$, beyond point A_1) shifted to point B_1 .

all located above the ones obtained for model B , suggesting that the presence of financial frictions moves the economy to regions with higher unemployment levels and therefore higher welfare costs. Furthermore, since the welfare cost function shifts upwards, the business cycle starts being desirable ($\tau > 0$) only for large expansions (for a technological change of at least 1.15, beyond point A_1 in figure 5). Even in the case of large expansions, the slope of the welfare cost function is nearly flat: welfare gains remain very small. In that sense, we argue that the presence of financial frictions makes the welfare costs more asymmetric than in the matching model. To illustrate this point, we report, in panel (d) of figure 5, the welfare cost function of model A (just the segment beyond point A_1 , with $\tau < 0$) at point B_1 . It can then be easily seen that, when both models predict welfare gains (expansions), the welfare gains

remain small in model A, with financial frictions, while they increase quasi-linearly without financial frictions, in model B.

6 Conclusion

This paper provides a quantitative assessment of welfare costs of fluctuations in a labor market search model with financial frictions à la Kiyotaki & Moore (1997). Because of labor market search frictions, fluctuations generate higher average unemployment rate with respect to its steady-state value, increasing the welfare cost of fluctuations. Financial frictions amplify this mechanism, together with the associated welfare costs. We show that the business-cycle costs are sizable: they are 50 times greater than in Lucas' estimate. Without financial constraints, the magnitude of the business cycle costs is reduced to 6 times Lucas' evaluation. Our model also allow us to obtain a large responsiveness of the job finding rate to the business cycle. Indeed, financial frictions entail wage sluggishness that helps the model match the large changes in job finding rates observed in the data; at the same time, it preserves the real wage volatility observed in the data.

We discuss how the presence of financial frictions sharply increases the welfare cost of fluctuations. This result suggests that policies aiming at removing financial frictions have also significant stabilizing macroeconomic effects. Moreover, our paper recovers the existence of significant asymmetries in the response of welfare to business cycles. Therefore, the study of stabilization policies must take into account the presence of non-linearities. Policy analysis by Kreps & Scheffel (2014) and, Albertini & Poirier (2014) or Roulleau-Pasdeloup (2014) explore promising avenues.

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Appendix

A Data

Aggregate data: The following quarterly time series come FRED database, from the Federal Reserve Bank of Saint Louis' website (1976Q1-2013Q1). y is Real Gross Domestic Product from the FRED database (mnemonicsGDPC96) divided by the Civilian Non institutional Population from the FRED database (mnemonics CNP16OV). c is Real Personal Consumption Expenditures from the FRED data-base (mnemonics PCECC96) divided by the Civilian Non institutional Population from the FRED database (mnemonics CNP16OV)

Labor market data: w is Compensation of Employees: Wages & Salary Accruals from the FRED database (mnemonics WASCUR) divided by Civilian Employment (CE16OV). N is Civilian Employment (CE16OV) divided by Civilian Non institutional Population. U is FRED, Civilian Unemployment Rate (UNRATE), Percent, quarterly, Seasonally Adjusted. The previous time series are taken from the FRED database. As for the time series of job finding rate, we use monthly CPS data from January 1976 to March 2013. We follow all the steps described in Shimer (2012). As in Shimer (2012), we correct for time aggregation and take quarterly averages of monthly observations. V are vacancies Total Nonfarm, Total US Job Openings JTS00000000JOL, Seasonally Adjusted Monthly data from BLS. We take quarterly averages of this time series that is available only from December 2000 onwards.

Debt: We follow Jermann & Quadrini (2012). Financial data come from the Flow of Funds Accounts of the Federal Reserve Board. The debt stock is constructed using the cumulative sum of net new borrowing measured by the 'Net increase in credit markets instruments of non financial business'³⁷. Since the constructed stock of debt is measured in nominal terms, it is deflated by the price index for business value added from NIPA. The initial (nominal) stock of debt is set to 94.12, which is the value reported in the balance sheet data from the

³⁷Nonfinancial business; credit market instruments; liability; Net increase in credit markets instruments of non financial business, millions of dollars (nominal). FA144104005.Q, F.101 Line 28,

Flow of Funds in 1952.I for the nonfarm non financial business. The cumulative sum starts in 1952, which, as in Jermann & Quadrini (2012), is not likely to affect our data starting on January 1976. R is the log of 1+ the Bank Prime Loan Rate (MPRIME) (used as a reference for short-term business loan) from the FRED database.

Cyclical components of the data: All data are quarterly (from 1976:Q1 through 2013:Q1), in logs, $HP(\lambda = 1600)$ filtered and multiplied by 100 in order to express them in percent deviation from steady state. Ψ is the job finding rate computed from Monthly CPS data from January 1976 to March 2013 using Shimer (2012)'s methodology. It measures the probability for an unemployed worker to find a job. As for financial data on debt and interest rate, we follow Jermann & Quadrini (2012). We finally check that our financial and labor market time series are consistent with the data available on line for Shimer (2012) and Jermann & Quadrini (2012).

B Model

B.1 Household

Each household knows that the evolution of S follows (3), so that (5) can be written as:

$$N_t = (1 - s)N_{t-1} + \Psi_t(1 - (1 - s)N_{t-1}) \quad (18)$$

The dynamic problem of a typical household can be written as follows

$$\mathcal{W}(\Omega_t^H) = \max_{C_t^n, C_t^u, B_t} \{N_t U(C_t^n) + (1 - N_t)U(C_t^u + \Gamma) + \mu E_t \mathcal{W}(\Omega_{t+1}^H)\}$$

subject to (18) and (6), given the initial conditions on state variables (N_0, K_0, B_0) and $\Omega_t^H = \{N_{t-1}, \Psi_t, w_t, b_t, T_t, B_{t-1}\}$, the vector of variables taken as given by households. Let λ_t be the shadow price of the budget constraint. The first order conditions associated with

consumption choices are

$$(C_t^n)^{-\sigma} = (C_t^u + \Gamma)^{-\sigma} = \lambda_t$$

Hence $\tilde{U}_t^n = \tilde{U}_t^u$. The first order condition associated to bond holdings reads:

$$-\lambda_t + \mu E_t [R_t \lambda_{t+1}] = 0 \quad (19)$$

B.2 Entrepreneur

The firm's program is

$$\begin{aligned} \mathcal{W}(\Omega_t^F) &= \max_{C_t^F, L_t, B_t, V_t, N_t} \{U(C_t^F) + \beta E_t [\mathcal{W}(\Omega_{t+1}^F)]\} \\ \text{s.t.} &\left\{ \begin{aligned} -C_t^F - R_{t-1}B_{t-1} - q_t [L_t - L_{t-1}] - w_t N_t - \bar{\omega} V_t \\ &\quad + Y_t(A_t, L_{t-1}, N_t) + B_t = 0 \quad (\lambda_t^F) \\ -B_t - \bar{\omega} V_t + m E_t [q_{t+1} L_t] &= 0 \quad (\lambda_t^F \varphi_t) \\ -N_t + (1-s)N_{t-1} + \Phi_t V_t &= 0 \quad (\xi_t) \end{aligned} \right. \end{aligned} \quad (20)$$

given the initial conditions N_0, B_0 , where $\Omega_t^F = \{N_{t-1}, \Psi_t, w_t, b_t, \pi_t, T_t, B_{t-1}, L_{t-1}\}$ is the vector of variables taken as given by firms. Letting $\lambda_t^F, \lambda_t^F \varphi_t$, and ξ_t be the Lagrange multipliers associated to (8), (10) and (11) the first order conditions of problem (20) read:

$$U'(C_t^F) = \lambda_t^F \quad (21)$$

$$\lambda_t^F q_t = \beta E_t \left[\lambda_{t+1}^F \left(q_{t+1} + \frac{\partial Y_{t+1}}{\partial L_t} \right) \right] + \lambda_t^F \varphi_t m E_t [q_{t+1}] \quad (22)$$

$$(1 - \varphi_t) \lambda_t^F = \beta E_t \lambda_{t+1}^F R_t \quad (23)$$

$$\xi_t = \lambda_t^F \bar{\omega} \frac{(1 + \varphi_t)}{\Phi_t} \quad (24)$$

$$\xi_t = \lambda_t^F \left[\left(\frac{\partial Y_t}{\partial N_t} \right) - w_t \right] + (1-s) \beta E_t [\xi_{t+1}] \quad (25)$$

where (21) is the condition associated to consumption and (24) the one on vacancy posting³⁸. Equation (22) is the one associated to land accumulation. It implies that, in equilibrium,

³⁸Note that, entrepreneurs are not risk neutral. By letting $\lambda_t^F = 1$ we recover the canonical search model.

the value of current marginal utility of consumption needs to equal the indirect value of utility deriving from land accumulation, i.e.: *i)* the value of future consumption utility deriving from reselling land in the next period, $\beta E_t \lambda_{t+1}^F q_{t+1}$; *ii)* the future consumption utility arising from the product of land, $\beta E_t \lambda_{t+1}^F \frac{\partial Y_{t+1}}{\partial L_t}$; *iii)* the additional utility arising from current consumption related to the effect of land in loosening the collateral constraint, $\varphi_t m \lambda_t^F E_t [q_{t+1}]$.

Equation (23) is a modified Euler equation. When the collateral constraint is not binding, φ_t is equal to zero and we recover the standard Euler equation. When the debt limit is binding, $\varphi_t > 0$ and $\varphi_t = 1 - \beta \frac{E_t \lambda_{t+1}^F R_t}{\lambda_t^F}$ implying that firms' marginal utility of current consumption is greater than their discounted marginal utility of future consumption. Impatient firms choose thus to increase consumption up to the limit imposed by (10).

B.3 The wage curve

From the household's intertemporal program, one gets:

$$\begin{aligned} \mathcal{V}_t^H &= \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_{t-1}} = \frac{\partial \mathcal{W}(\Omega_t^H)}{\partial N_t} \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}} \\ &= [U_t^n - U_t^u + \lambda_t w_t - \lambda_t b_t - \lambda_t (C_t^n - C_t^u)] \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}} \end{aligned}$$

With $C_t^n = C_t^u$ and $U_t^n = U_t^u$, we have

$$\mathcal{V}_t^H = [\lambda_t w_t - \lambda_t b_t] \frac{\partial N_t}{\partial N_{t-1}} + \mu E_t \left(\frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \frac{\partial N_t}{\partial N_{t-1}}$$

Where, from (18) $\frac{\partial N_t}{\partial N_{t-1}} = (1-s)(1-\Psi_t)$, so that

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s)(1-\Psi_t) \left[w_t - b_t + \mu E_t \left(\frac{1}{\lambda_t} \frac{\partial \mathcal{W}(\Omega_{t+1}^H)}{\partial N_t} \right) \right] \quad (26)$$

From the firms' program $\mathcal{V}_t^F = \frac{\partial \mathcal{W}(\Omega_t^F)}{\partial N_{t-1}} = \xi_t (1-s)$ where $\xi_t = \lambda_t^F \bar{\omega} \frac{(1+\varphi_t)}{\Phi_t}$, thus:

$$\begin{aligned} \frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} &= (1-s) \frac{\bar{\omega}}{\Phi_{t+1}} \lambda_{t+1}^F (1+\varphi_{t+1}) \\ \frac{\mathcal{V}_t^F}{\lambda_t^F} &= (1-s) \frac{\bar{\omega}}{\Phi_t} (1+\varphi_t) \end{aligned}$$

Then, using (17) we obtain:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s) \left[\frac{\partial Y_t}{\partial N_t} - w_t (1+\varphi_t) + \beta E_t \left(\frac{1}{\lambda_t^F} \frac{\partial \mathcal{W}(\Omega_{t+1}^F)}{\partial N_t} \right) \right]$$

Therefore, the surpluses are, respectively:

$$\frac{\mathcal{V}_t^F}{\lambda_t^F} = (1-s) \left[\frac{\partial Y_t}{\partial N_t} - w_t + \beta E_t \left(\frac{\lambda_{t+1}^F}{\lambda_t^F} \frac{\mathcal{V}_{t+1}^F}{\lambda_{t+1}^F} \right) \right] \quad (27)$$

$$\frac{\mathcal{V}_t^H}{\lambda_t} = (1-s) (1-\Psi_t) \left[(w_t - b_t) + \mu E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{\mathcal{V}_{t+1}^H}{\lambda_t} \right) \right] \quad (28)$$

By maximizing (12) with respect to the wage, we obtain $\left(\frac{\mathcal{V}_t^H}{\lambda_t} \right) = \left(\frac{\mathcal{V}_t^F}{\lambda_t^F} \right) \frac{(1-\epsilon)(1-\Psi_t)}{\epsilon}$. By substituting for (27) and (28), and rewriting it, we obtain the wage curve

B.4 Proof of proposition 1

In equilibrium, the *JC*, equation (16) and the *WC*, equation (14) need to intersect, i.e.:

$$\begin{aligned} &MPL + \theta^{1-\psi} \frac{\bar{\omega}}{\chi} \left(1 + 1 - \frac{\beta}{\mu} \right) [(1-s)\beta - 1] \\ &= \epsilon O + (1-\epsilon) MPL + (1-s)(1-\epsilon) \bar{\omega} \mu \left(1 + 1 - \frac{\beta}{\mu} \right) \left[\frac{\theta^{1-\psi}}{\chi} (\beta - \mu) + \mu \theta \right] \end{aligned} \quad (29)$$

where we $\frac{\partial Y}{\partial N} \equiv MPL$ and we denote by O the outside option. For simplicity, let as proxy the impatience gap with the steady-state level of the Lagrange multiplier associated to the collateral constraint, $\bar{\varphi} = 1 - \frac{\beta}{\mu}$. Indeed, $\bar{\varphi}$ is increasing in the impatience gap. Equation

(29) can be rewritten as:

$$\epsilon(MPL - O) = (1 + \bar{\varphi}) \bar{\omega} \left[(1 - s)(1 - \epsilon)\mu\theta + \frac{\theta^{1-\psi}}{\chi} (1 - (1 - s)\mu(1 - \bar{\varphi}\epsilon)) \right] \quad (30)$$

Let us also define $g(\theta, \bar{\varphi})$ as

$$g(\theta, \bar{\varphi}) = (1 + \bar{\varphi}) \bar{\omega} \left[(1 - s)(1 - \epsilon)\mu\theta + \frac{\theta^{1-\psi}}{\chi} (1 - (1 - s)\mu(1 - \bar{\varphi}\epsilon)) \right]$$

Note that $g(\theta, \bar{\varphi})$ is increasing in θ , since

$$g'_{\theta}(\theta, \bar{\varphi}) = (1 + \bar{\varphi}) \bar{\omega} \left[(1 - s)(1 - \epsilon)\mu + (1 - \psi) \frac{\theta^{-\psi}}{\chi} (1 - (1 - s)\mu(1 - \bar{\varphi}\epsilon)) \right] > 0$$

Moreover, $g(\theta, \bar{\varphi})$ is concave in θ since

$$g''_{\theta}(\theta, \bar{\varphi}) = (1 + \bar{\varphi}) \bar{\omega} \left[-\psi(1 - \psi) \frac{\theta^{-\psi-1}}{\chi} (1 - (1 - s)\mu(1 - \bar{\varphi}\epsilon)) \right] < 0$$

In addition, $\forall \bar{\varphi}$, we have

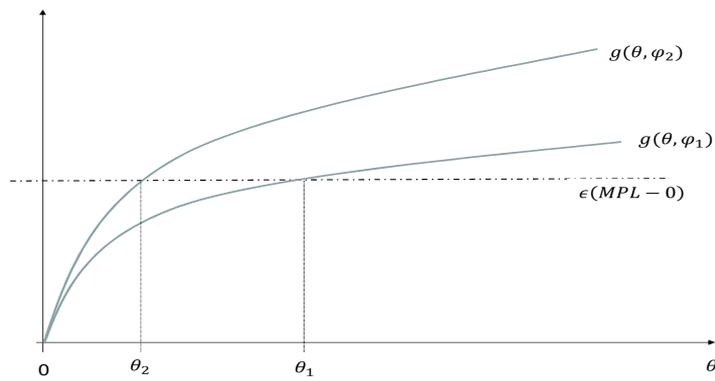
$$\lim_{\theta \rightarrow +\infty} g(\theta, \bar{\varphi}) = +\infty$$

and $g(0, \bar{\varphi}) = 0$. Finally, $g(\theta, \bar{\varphi})$ is steeper in an economy with financial frictions, indeed:

$$\frac{\partial g'(\theta, \bar{\varphi})}{\partial \bar{\varphi}} = \bar{\omega} \left[\begin{array}{l} (1 - s)(1 - \epsilon)\mu + (1 - \psi) \frac{\theta^{-\psi}}{\chi} (1 - (1 - s)\mu(1 - \bar{\varphi}\epsilon)) \\ + (1 + \bar{\varphi})(1 - \psi) \frac{\theta^{-\psi}}{\chi} (1 - s)\mu\epsilon \end{array} \right] > 0$$

Hence, labor market tightness is lower in case of financial frictions (On figure 6, $\theta_2 < \theta_1$ with $\varphi_2 > \varphi_1$). As no restrictions on parameter values are needed, this result is not ambiguous. This means that the equilibrium level of θ is eventually driven by the steeper JC curve.

Figure 6: Steady state labor market tightness



Labor market tightness is lower in case of financial frictions : $\theta_2 < \theta_1$ with $\varphi_2 > \varphi_1$